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TEXT-BOOK OF ARITHMETIC,

FOR THE USE OF TEACHERS;

INCLUDING

ALL THE EXERCISES IN THE PUPILS' MANUAL OF ARITHMETIC.

BY THE

REV. JOHN HUNTER, M.A.,

FORMERLY OF THE NATIONAL SOCIETY'S TRAINING COLLEGE, BATTERSEA;
AUTHOR OF THE 'TEXT-BOOK OF ENGLISH GRAMMAR,' ETC.

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ORIGINAL DEDICATION.

TO

THE VENERABLE

JOHN SINCLAIR, A.M.,

ARCHDEACON OF MIDDLESEX,

IN ADMIRATION OF THE GREAT SERVICE HE HAS RENDERED TO THE CAUSE OF ELEMENTARY EDUCATION, AND IN GRATEFUL ACKNOWLEDGMENT OF MANY PERSONAL FAVOURS,

THE TEXT-BOOK AND MANUAL OF ARITHMETIC,

OWING SOME OF THEIR PECULIAR FEATURES TO HIS VALUED SUGGESTIONS, ARE MOST RESPECTFULLY INSCRIBED BY HIS OBLIGED AND OBEDIENT SERVANT,

THE AUTHOR.

PREFACE.

THE Author of this little volume presumes that it will prove acceptable to the numerous teachers who have placed his MANUAL of ARITHMETIC in the hands of their pupils, all the Exercises of the Manual being here incorporated with the demonstrative and explanatory text, which formed the First Part of the Companion and Supplement. In the revision of that text great care has been exercised, and many important improvements have been effected; and it is conceived that the principal Rules of Arithmetic are now presented, in this publication, with no unnecessary tediousness, yet with such strictness of reasoning and such simplicity and fulness of exposition, as shall amply meet the requirements of those who may employ it as a medium of self-instruction, or as a guide for the instruction of others.

J. H.

Uxbridge School,
May, 1852.

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ARITHMETIC.

ARITHMETIC teaches the nature of numbers, and how to increase, diminish, and compare them.

Its processes are usually conducted by means of convenient marks that stand as signs for numbers.

Before the pupil's attention is directed to the *figures* commonly employed for the notation of numbers, he may be taught the nature of the operations termed Addition, Subtraction, Multiplication, and Division, by reference to a row of ten units, or counters, chalked on a black board. We proceed to indicate a form of instruction for this purpose, recommending the teacher to refer with a pointer to the row of units, while he gives the lessons, and to encourage the pupils to name the sum, remainder, &c., as he goes along. Promiscuous examination should follow each lesson, first with the aid of the counters, and afterwards without it.

INTRODUCTORY EXERCISES.

ROW OF UNITS: | | | | | | | | | |

A single thing is referred to by the number *One*.

ADDITION.—*One* together with another *one* makes the number *two*; *two* together with *one* makes *three*; *three* together with *one* makes *four*; and so on, until *ten* is the sum attained.

One together with *two* makes *three*; *two* together with *two* makes *four*; *three* together with *two* makes *five*; &c.

One together with *three* makes *four*; *two* with *three* makes *five*; *three* with *three* makes *six*; &c.

Examination.—*Four* apples and *one* apple make how many? Add together *three* and *two*. Add four and *one* and *three*. What is the sum of two pennies and

five pennies? Put into one sum *three* and *three* and *four*.

SUBTRACTION.—*Ten* when *one* is taken from it becomes *nine*; *nine* lessened by *one* is *eight*; *eight* lessened by *one* is *seven*; &c.

Ten lessened by *two* becomes *eight*; *nine* lessened by *two* is *seven*; &c.

Examination.—From *nine* take *five*, what is the number left? From *seven* take *two*, what remains? *Four* and what make *ten*? Subtract *nothing* from *eight*; *three* from *eight*. What three numbers make up *ten*?

MULTIPLICATION.—*One* taken *one* time, or *once*, is *one*; *one* taken *two* times, or *twice*, is *two*; *one* taken *three* times is *three*; &c.

Two taken *once* is *two*; *two* taken *twice* is *four*; *two* taken *thrice* is *six*; &c.

Examination.—*One* and *one* and *one* are how many times *one*? How many are *three* times *two*? *Four* times *one* and *three* times *one*? Multiply *five* by *two*; *two* by *five*. Take *two* from *three* times *two*. Take *four* from *five* times *one*. Take *four* from *five* times *two*. When I say, Add *one* to *three* times *two*, how many operations are proposed? Their names? When I say, From *four* times *one* take *two*, what two operations are proposed? Subtract *ten* from *five* times *two*.

DIVISION.—*One* is contained in the number *ten*, *ten* times; *one* is contained in *nine*, *nine* times; *one* is contained in *eight*, *eight* times; &c.

Two is contained in *ten five* times; *two* is contained in *nine*, *four* times (with *one* over); &c.

Examination.—How often is *five* contained in *ten*? in *seven*? in *five*? in *three*? Divide *eight* by *four*; *five* by *one*; *ten* by *six*. If *six* is divided into *two*

equal parts, what is the value of each part? What is the *third* part of *nine*? the *tenth* part of *ten*? the *half* of *eight*? the *fifth* of *ten*? How many *tenth* parts of *ten* make the number *four*? How many *third* parts of *six* make the number *eight*? How many *third* parts of *nine* make the number *nine*? Add *four* to the *half* of *four*. Subtract *one* from the *fifth* of *five*. Multiply *three* by the *fourth* part of *eight*; by the *ninth* of *nine*.

I.—NOTATION AND NUMERATION.

NOTATION means writing figures to represent values.

NUMERATION means naming the values which figures represent.

The figures 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, are convenient signs, used instead of the words *nothing*, *one*, *two*, *three*, *four*, *five*, *six*, *seven*, *eight*, *nine*; and by means of these figures any number may be expressed.

A single figure, or the right-hand figure of a number, represents so many *tenth-parts* of *ten*, or *simple units*, or *ones*; the second figure from the right signifies so many units of *ten*, or *tens*; the third so many units of *tens* of *ten*; the fourth so many units of *tens* of *tens* of *ten*. Thus—

7 is seven *simple units*, or *tenth-parts* of *ten*, or *ones*;

70 is seven *tens*, and no *simple units*;

63 is six *tens*, and three *simple units*;

428 is four *tens* of *ten*, two *tens*, and eight *ones*;

3505 is three *tens* of *tens* of *ten*, five *tens* of *ten*, and five *ones*.

All whole numbers, therefore are represented by figures which express so many *tenth parts* of *ten*, or so many *whole tens*, or so many combined *tens* and *tenths* of *ten*.

In naming the value of a number, of more than one figure, the usual method is to read it from left to right.

The name *tenth-parts* of *ten*, or *simple units*, or *ones*, is generally not mentioned. Thus, 7 is simply called *seven*.

The figures representing simple *tens* are named in the following manner: 10 ten, 20 twenty, 30 thirty, 40 forty, 50 fifty, 60 sixty, 70 seventy, 80 eighty, 90 ninety.

The figures representing *tens* of *ten* are usually called *hundreds*. Thus: 100 one hundred, 200 two hundred, &c.

The figures representing *tens* of *tens* of *ten* (or, *tens* of *a hundred*) are usually called *thousands*. Thus: 1000 one thousand, 2000 two thousand, &c.

The numbers between *ten* and *twenty* are represented and named thus: 11 eleven, 12 twelve, 13 thirteen, 14 fourteen, 15 fifteen, 16 sixteen, 17 seventeen, 18 eighteen, 19 nineteen.

The teacher should now illustrate the foregoing principles and usages by promiscuous examples; and should ascertain by examination that the pupil thoroughly understands the scheme of notation so far as thousands; it is, for the present, inexpedient to exceed that limit.

Promiscuous Examples.—The first number consists of 3 tens, and 5 tenths of ten; and it is named *thirty-five*.

The second number consists of 3 tens of ten, 5 simple tens, and no tenths of ten; and it is named *three hundred and fifty*.

&c. &c.

35
350
472
601
717
2748
5060
9911

It may now be more distinctly stated,—

That every figure denotes units *ten times* as great in value, by being shifted to the place next it on the left:—Compare 32 and 23, for instance.

That, *therefore*, nine significant figures, together with *zero*, or the cipher 0, are sufficient to express all values.

That the cipher, which has by itself no value, is used to mark the place where any value is wanting, in order

to secure a proper local value for the figure or figures on its left. Compare, for instance, 30 with 3.

That values ascending from *zero* to *nine* (whether they mean so many simple units, or so many units of ten, &c.) are represented by *different figures;* and that *ten*, the next higher value, is represented by a *difference of position.*

When the following Numeration and Notation Exercises have been satisfactorily performed, the pupil should be prepared (by such instruction as is given in next section) to work the first forty Exercises in Simple Addition; and after performing these, he may be taught, by reference to the annexed Table, the numeration of values higher than thousands.

NUMERATION TABLE.

Billions	Hundreds	Tens	*Thousands*	Hundreds	Tens	Millions	Hundreds	Tens	*Thousands*	Hundreds	Tens	Ones
5;	6	2	9,	1	4	4;	2	0	7,	0	3	8

EXERCISES IN NUMERATION.

27; 72; 30; 16; 61; 59; 80; 15; 126; 621; 365; 900; 420; 512; 111; 402; 610; 707; 556; 819; 918; 106; 2154; 5678; 3000; 6660; 4602; 4062; 8005; 1234; 9013.

EXERCISES IN NOTATION.

Thirty-three; Forty-seven; Ninety; Sixty-nine; Fourteen; Twenty; Twelve; Five hundred; Three hundred and seventy-eight; Four hundred and sixteen; One hundred and ten; Five hundred and three; Eight hundred and seventy-six; Two hundred and forty; Three hundred and three; Two thousand six hundred and forty-nine; Four thousand three hundred and thirty-four;

Five thousand one hundred and ninety; Nine thousand; Eight thousand and eighty-six; Seven thousand five hundred and four; Six thousand seven hundred and twenty-four; One thousand and ten; Two thousand and seven.

II.—SIMPLE ADDITION.

Exemplification.—Suppose I have 3 shillings, and you give me 5 shillings more, I shall then have 8 shillings; if I get 4 shillings besides, I shall have 12 shillings. Here I have increased the number 3 by combining it with 5, and further increased it by combining it also with 4.

Definition.—When a number is increased by combining it with another, or with several others, so as to form one Amount or Sum, the process is called ADDITION.

Before slate-exercises in Addition have been entered upon, and occasionally afterwards, the pupil should be practised in the mental addition of simple units of which the final amount does not exceed 100.

Addition is often signified by the mark +, which is called the sign of addition, and named *plus* (a Latin word which denotes *more*). We may, accordingly, represent the addition of 3, 5, and 4, by writing 3 + 5 + 4 equal to 12; or, employing another mark of operation, called the sign of equality, 3+5+4 = 12, which is read 3 *plus* 5 *plus* 4 *equal to* 12.

Let the pupil be required to represent by the above signs of operation, the addition of two or more of the first twelve Exercises.

When several large numbers are proposed for addition, they are usually arranged in successive lines, all the simple units forming one column, the tens another column, &c.

For example, to find what sum the following expression is equal to; 38 + 59 + 47 + 26 + 75:—

Let the given numbers be arranged as in the margin; and let the columns of ones and tens be added separately. It is usual to commence at the bottom, and to add upwards; although the sum might be found by any order of addition. Add, then, first the ones; 5 and 6 make 11, and 7 more make 18, and 9 more make 27, and 8 more make 35 ones, which is the sum of the first or right-hand column. Now 35 ones consist of 3 units of ten and 5 units of one; therefore, write 5 under the column of ones, and combine the three tens with the column of tens; thus, 3 and 7 are 10, and 2 are 12, and 4 are 16, and 5 are 21, and 3 are 24 tens; that is 2 tens of ten and 4 simple tens, or 2 hundreds and 4 tens; therefore the whole sum is 245.

Tens.	Ones.
3	8
5	9
4	7
2	6
7	5
2	4 5

For illustration's sake, the above example may be exhibited also in the following manner:—

	38	=	3	tens	+	8	ones		
	59	=	5	tens	+	9	ones		
	47	=	4	tens	+	7	ones		
	26	=	2	tens	+	6	ones		
	75	=	7	tens	+	5	ones		
			21	tens	+	35	ones		
But	21	tens	=	2	hundreds	+	1	ten	
	35	ones	=				3	tens	+ 5 ones
Ans.	245	units	=	2	hundreds	+	4	tens	+ 5 ones

The number found by Addition is called the *sum*, *amount*, or *aggregate*; and the numbers added are called the parts of the aggregate, or the *aggregate parts*.

Rule.—Place the given numbers so that all the ones may form one column, all the tens another, &c. Find the sum of the column of ones, and, dividing that sum into tens and ones, set down the ones at the foot of their column, and carry the tens, if any, to the column of tens. Proceed similarly with the remaining columns, and under the left-hand column write its entire sum.

The *Reason* of the Rule of Addition is simply the axiom, that any whole is equal to the sum of its aggregate parts.

As a good method of *proving* the Pupil's accuracy in the ordinary process, he may be required to begin at the top of the left-hand column, and add downwards. Let the carrying figures be reserved in a second line, as in the margin; and finally, add together the two lines thus produced. The ordinary method of proving Addition, by reserving the head number for combination with the sum of the remaining numbers, does not secure the Pupil against the recurrence of mistakes.

38
59
47
26
75
—
245
—
15
23
—
245

The teacher should not be content with ascertaining that the pupils have worked the Exercises in Addition, but should occasionally call upon them, after they have silently performed some particular Exercise, to go over the steps of the calculation audibly, and to show that they understand the numeration of the figures. This suggestion applies to all the elementary rules.

EXERCISES.

1.	2.	3.	4.	5.	6.	7.	8.	9.	10.	11.	12.
2	3	5	3	6	4	7	5	5	6	6	5
3	5	6	8	3	6	8	9	7	1	7	9
5	2	4	6	4	8	9	7	9	9	8	6
2	6	5	3	8	4	6	3	6	8	5	9
3	4	6	4	7	2	4	9	2	9	8	4
1	3	7	2	5	5	7	8	9	8	9	7
4	1	5	1	2	6	5	7	6	9	6	8
—	—	—	—	—	—	—	—	—	—	—	—

Answers.—1. 20; 2. 24; 3. 38;
4. 27; 5. 35; 6. 35;
7. 46; 8. 48; 9. 44;
10. 50; 11. 49; 12. 48.

13.	14.	15.	16.	17.	18.	19.	20.	21.	22.
26	65	37	45	68	96	97	64	59	77
32	48	49	60	79	78	89	96	7	89
24	26	56	26	85	41	6	28	85	56
18	72	20	49	26	86	94	9	67	74
36	57	84	54	97	75	18	76	59	98
27	38	18	75	68	40	5	10	6	84
43	84	74	33	79	57	56	57	8	57
74	48	47	64	97	29	43	8	49	86
69	37	50	89	58	68	95	9	84	64

23. What is the amount of 46+56+19+7+83+8+80?

24. Collect into one sum 24+37+48+5+6+17+97+36+89+8.

25.	26.	27.	28.	29.	30.	31.	32.
663	675	838	673	365	13	476	565
329	288	657	45	384	36	892	583
487	374	96	450	793	53	289	289
874	657	85	238	652	808	365	605
563	839	364	54	279	618	846	747
275	700	90	86	540	9	468	330
468	806	909	19	810	90	234	466
906	397	25	932	185	900	115	699

Answers.

13. 349; 14. 475;
15. 435; 16. 495; 17. 657;
18. 570; 19. 503; 20. 357;
21. 424; 22. 685; 23. 299;
24. 367; 25. 4565; 26. 4736;
27. 3064; 28. 2497; 29. 4008;
30. 2527; 31. 3685; 32. 4284.

33.	34.	35.	36.	37.	38.	39.	40.
679	407	828	738	345	587	987	634
856	645	587	861	678	293	456	703
365	874	965	975	901	896	876	658
492	758	876	686	234	689	345	49
249	986	494	798	567	978	789	876
783	869	77	859	890	568	567	87
534	697	89	964	123	743	432	96
827	530	709	573	456	896	678	89
448	671	78	591	789	608	766	86

41.	42.	43.	44.	45.	46.	47.
4368	7545	3819	4105	9876	5962	2468
2536	6457	6349	6309	2345	380	246
9875	3869	6857	2800	6789	596	3579
8394	4004	5686	7636	3456	3839	67
7743	8680	9795	5990	8765	7943	6070
3912	4738	8101	8578	5678	58	1357
5656	5967	4293	3754	4567	585	5146
9028	5579	3734	9787	5432	219	408

48. The first volume of a book contains 372 pages, the second 400, the third 382, the fourth 389, and the fifth, or last volume, 375. How many pages does the work contain?

49. I have 83 apples, and my brother has 64;—if I should get 59 added to my number, and my brother 32 added to his; how many should we have together?

50. Charles is 8 years old; John is 6 years older than Charles; William is 8 years older than John; and Henry is 7 years older than William. What is the sum of all their ages?

Answers.

33. 5233; 34. 6437; 35. 4703; 36. 7045; 37. 4983; 38. 6258; 39. 5896; 40. 3278; 41. 51512; 42. 46839; 43. 48634; 44. 48959; 45. 46908; 46. 19582; 47. 19341; 48. 1918; 49. 238; 50. 73.

51.	52.	53.	54.	55.	56.
3568	6957	4896	63827	78901	83543
2978	8593	5637	63591	23456	56280
4769	7632	3795	97486	89012	47967
6205	5858	7854	56935	34567	5839
7667	9787	3977	86758	90123	365
9006	8969	8589	49640	45678	4694
5854	4376	6006	85379	99999	88080
4968	5448	9668	64862	68068	9756
3496	4556	4325	37001	33786	319

57. Perform the addition of 679 + 868 + 996 + 849 + 886 + 8979 + 798 + 7689 + 937 + 978 + 986 + 3869 + 688.

III.—SIMPLE SUBTRACTION.

Exemplification.—If I have 8 nuts, and give away 2 of them, I shall have 6 remaining.

If I have 10 apples, and you have 6 apples, I have 4 apples more than you. I thus compare my number with yours; and, by supposing 6 of my apples to be taken away, I observe that I should have 4 left; whereas, if 6 of your apples were taken away, you should have none left. I have, therefore, 4 more than you.

Definition.—When a number is to have part of it taken away, the process is called SUBTRACTION.

Before slate-exercises in this Rule, let the pupil be practised in mental subtractions.

Subtraction is often indicated by the mark −, which is named *minus* (a Latin word which signifies *less*). We may, accordingly, represent the subtraction of 9 from 13 by writing 13 − 9 = 4, which is read 13 *minus* 9 equal to 4.

Answers. 51. 48511; 52. 62176; 53. 54747; 54. 605479; 55. 563590; 56. 296843; 57. 29202.

When two large numbers are given, for the subtraction of the one from the other, it is usual to place the greater over the less, so that ones may be subtracted from ones, tens from tens, &c., separately.

For example, let it be required to find what single number is equal to 8956 – 452.

Placing the less number under the greater, as in the margin, we say 2 ones from 6 ones leaves 4 ones; 5 tens from 5 tens leaves 0 tens; 4 hundreds from 9 hundreds leaves 5 hundreds; 0 thousands from 8 thousands leaves 8 thousands. Therefore the entire remainder is 8504.

Thous.	Hund.	Tens.	Ones.
8	9	5	6
	4	5	2
8	5	0	4

When the pupil has performed the first five Exercises, his attention may be called to an Example such as the following:—

Subtract 379 from 862.

Here, in proceeding to subtract 9 ones, we perceive that the 2 ones do not admit such subtraction; but by taking 1 of the 6 tens with the 2, and saying 9 from 12 leaves 3, then we should have 5 tens instead of 6 for the next subtraction, and taking, with these 5, 1 of the 8 hundreds, we may say 7 tens from 15 tens, leaves 8; and, lastly, 3 hundreds from 7 hundreds leaves 4 hundreds.

862	
379	
483	

Thus, when an upper figure does not allow subtraction, borrowing 1 from the left will answer the purpose, if we remember that the value on the left is afterwards to be reckoned 1 less.

This *natural* method, as it may be called, being inconvenient, is supplanted, in common use, by an *artificial* method, founded on the circumstance that two numbers, if equally increased, preserve the same difference, because the numbers added have no difference. Thus 9 and 4 have a difference of 5; add 3 to each, and we have 12 and 7, with the same difference of 5.

If, accordingly, in subtracting 379 from 862, we increase each number by 10 *ones* or 1 *ten*, we shall not alter the difference. Now, this increase can be given to

the 2 of the upper line, making it 12 ones, and to the 7 of the under line, making it 8 tens, thus making sure that the simple units shall admit of subtraction; for this makes—

the upper number consist of 8 hundreds 6 tens 12 ones,
and the lower number....3 hundreds 8 tens 9 ones.

But though the *ones* are now practicable for subtraction, the *tens* are not. Let us, therefore, add 10 *tens* to the upper, and 1 *hundred* to the lower quantity, and this will make the upper number consist of 8 hund. 16 tens 12 ones,
and the lower number .....4 hund. 8 tens 9 ones.

The difference, therefore, is 4 hund. 8 tens 3 ones.

Accordingly, whenever the value of a figure in the upper line is greater than that of the figure to be subtracted, conceive 10 to be added to the upper, and 1 to be added to the next lower.

The number to be lessened is called the *minuend;* the number to be subtracted, the *subtrahend;* and that which is left, the *remainder*, *difference*, or *excess.*

RULE.—Place the two given numbers as in Addition, the less under the greater. Beginning at the place of ones, subtract each lower figure from that above it, if possible; if not, add ten to the upper figure before subtracting, and carry one to the next figure of the subtrahend.

The *Reason* of the Rule of Subtraction is obviously implied in that of the Rule of Addition.

To *prove* the operation, add the remainder to the subtrahend; the sum should be equal to the minuend. The truth of this is manifest in the fact, that a portion taken away from a quantity, and the portion remaining, must, when recombined, make up that quantity.

The above example may be stated by means of the signs of operation: thus, $862 - 379 = 483$; and the pupil may very easily, and with advantage, be taught to vary the relations of the numbers without destroying the equation; thus $862 = 483 + 379$, which represents the method of proof; also $862 - 483 = 379$. These variations should be introduced to the pupil's notice, and exemplified by him, when he arrives at Ex. 29th.

It will be observed that Exercises 39 to 43 have the minuend placed under the subtrahend. As such an arrangement is often occasioned in the solution of problems, it may be expedient for the pupil to practise a little subtraction in that way.

EXERCISES.

1.	2.	3.	4.	5.
486957	549678	339458	8797869	6305421
243434	231226	203443	7521841	1001211

6.	7.	8.	9.
62695784	58493675	682096473	833679584
27048321	25628410	134030653	825758728

10.	11.	12.	13.
78964325	65697241	234165728	413945273
34972655	55891407	24559306	80988918

14.	15.	16.	17.	18.
437230	110347	305024	501365	930011
129260	19858	98569	4356	690033

19. Subtract *Fifty-one thousand six hundred and three* from *One hundred and seventy-one thousand one hundred and eighty.*

Answers. 1. 243523; 2. 318452; 3. 136015; 4. 1276028; 5. 5304210; 6. 35647463; 7. 32865265; 8. 548065820; 9. 7920856; 10. 45991670; 11. 9805834; 12. 209606422; 13. 332956355; 14. 307970; 15. 90489; 16. 206455; 17. 497009; 18. 239978; 19. 119577.

20. How much must be added to *Nine thousand three hundred and three* to make *One million?*

21.	22.	23.	24.
49372186	35420385	121754789	202350910
2615988	34626809	14790099	12780315

25.	26.	27.	28.
55544411	30000000	101010101	532602867
4648504	67104	2023032	531691969

29. 54321 − 12345;

30. 30063 − 466;

31. 7007 − 989;

32.	33.	34.	35.
54630710	10304352	214790035	100506301
320913	594806	890987	599383

36. Of 374 oranges, Harry's share is 159; Tom's is also 159. How many are still to be disposed of?

37. The present year is how many years later than the year 1066? and how many years earlier than the year 2000?

38. What single number is equal to 1630 + 946 − 809?

Answers. 20. 990697; 21. 46756198; 22. 793576; 23. 106964690; 24. 189570595; 25. 50895907; 26. 29932896; 27. 98987069; 28. 910898; 29. 41976; 30. 29597; 31. 6018; 32. 54309797; 33. 9709546; 34. 213899048; 35. 99906918; 36. 56; 38. 1767.

39.	40.	41.	42.	43.
281289	65396	2851	5863	4090
431057	130906	285130	606341	1102050

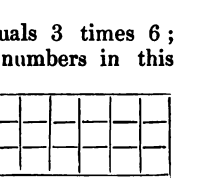
44. Add together 705 + 10328 + 8968 + 60079, and find how much the sum exceeds that of 8269 + 865 + 57.

45. Collect into one result 367 + 1898 − 562 + 57 + 957 + 562 − 673.

IV.—SIMPLE MULTIPLICATION.

Exemplification.—If I should buy 6 copy-books at 3 pence each, I should pay 6 times 3 pence, or 18 pence, altogether. Or if I should buy 3 copy-books at 6 pence each, I should pay 3 times 6 pence, or 18 pence altogether.

It appears, then, that 6 times 3 equals 3 times 6; and we can always interchange the numbers in this manner– as will be evident from the annexed diagram; for the 18 squares, of which it is composed, are made up of 3 rows of 6 squares each (*i. e*, 3 times 6 squares), or 6 rows of 3 squares each (*i. e.* 6 times 3 squares).



Definition.—Multiplication is a method of finding the sum of two or more equal numbers.

Make the pupil understand, from the above diagram, and by one or two numeral examples, that Multiplication is an expeditious method of producing the sum of several equal numbers; and that it requires the elementary sums which compose the

Answers. 39. 149768; 40. 65510; 41. 282279; 42. 600478; 43. 1097960; 44. 70889; 45. 2606.

Multiplication Table, to be carefully learned and readily recollected.

TABLE OF *Multiplication.*

2	times	2	are	4	4	times	4	are	16	7	times	7	are	49
		3	..	6			5	..	20			8	..	56
		4	..	8			6	..	24			9	..	63
		5	..	10			7	..	28			10	..	70
		6	..	12			8	..	32			11	..	77
		7	..	14			9	..	36			12	..	84
		8	..	16			10	..	40					
		9	..	18			11	..	44	8	times	8	are	64
		10	..	20			12	..	48			9	..	72
		11	..	22								10	..	80
		12	..	24	5	times	5	are	25			11	..	88
							6	..	30			12	..	96
3	times	3	are	9			7	..	35					
		4	..	12			8	..	40	9	times	9	are	81
		5	..	15			9	..	45			10	..	90
		6	..	18			10	..	50			11	..	99
		7	..	21			11	..	55			12	..	108
		8	..	24			12	..	60					
		9	..	27						10	times	10	are	100
		10	..	30	6	times	6	are	36			11	..	110
		11	..	33			7	..	42			12	..	120
		12	..	36			8	..	48					
							9	..	54	11	times	11	are	121
							10	..	60			12	..	132
							11	..	66					
							12	..	72	12	times	12	are	144

The sign of Multiplication is $\times$; thus, $5 \times 4 = 20$, signifies that 5 *multiplied by* 4 *equals* 20.

The mode of extending the application of the Multiplication Table is now to be exemplified.

Suppose we are required to multiply 3475 by 3.

This, of course, proposes to find the sum of 3475 repeated 3 times, and might be answered by addition; but the method by multiplication is more convenient, and is thus performed:—

Placing the number of times under the number to be repeated, we say, 3 times 5 ones are 15 ones, that is 1 ten and 5 ones; set down 5 ones and carry 1 ten; 3 times 7 tens are 21 tens, and 1 ten reserved makes

```
3475
   3
------
10425
```

22 tens; set down 2 tens, and carry 2 tens of tens, or hundreds; 3 times 4 hundreds are 12 hundreds, and 2 reserved hundreds make 14 hundreds; set down 4 hundreds, and carry 1 ten of hundreds, or thousand; 3 times 3 thousands are 9 thousands, and 1 thousand carried makes 10 thousands; set down 10 thousands entire.

The *reason* of this process is the same as for addition.

The sum 10425, when obtained by the process called Multiplication, is said to be the *product* of 3475; also, 3475 and 3 are called the *factors* (*i. e. producing* numbers), the upper factor being specially termed the *multiplicand* (*i. e.* the number *to be multiplied*), and the lower factor the *multiplier*.

In every arithmetical multiplication, the multiplier denotes simply *so many times* the multiplicand. Thus to give 6 *shillings* to each of 3 *men*, we require 3 *times* 6 shillings, or 6 shill. × 3; but we cannot say 3 *men times* 6 shillings; nor is it proper to write such an expression as 6 shill. × 3 men, unless merely by accommodation, to indicate whence the multiplier 3 is derived. For the purpose of arithmetical multiplication, the quantity, 3 men, has the notion of *men* abstracted or drawn away from it, leaving what is therefore called an *abstract* number. The notion of 3, when applied to and united with the notion of *men*, becomes 3 men, which is therefore called an *applicate*, or *concrete* (i. e. *compounded*) number.

Hence, it may be stated, that no number can be used as a multiplier except as an *abstract* number,—that the multiplicand *may* be a *concrete* number,—and that the product *must* be originally of the same name as the multiplicand.

When the pupil has performed the first twenty Exercises, and has proved his work, in some instances at least, by Addition, his attention may be directed to the following Examples:—

Examp. Multiply 478 by 10.

First, let the product be constructed by the usual successive steps; and then, by examination of the figures which are successively carried, it will appear, that to multiply any number by 10 requires only to annex 0 to the multiplicand. It is obvious, indeed, from the principle of our decimal notation, that annexing 0 at the *right* of the multiplicand, makes *each figure* of it ten

```
 478
  10
----
4780
```

times as great in value, and consequently gives an *entire product* ten times as great as the multiplicand.

Examp. Multiply 5274 by 30.

As 30 times is ten times *thrice*, we may first take 3 times the multiplicand, which gives 15822, and then take 10 times that product, which requires simply to annex 0.

5274
30
158220

By examination of the process it is seen that to multiply by an exact number of tens, we may multiply by the tens figure, as if it denoted ones, and then give the product a tenfold value by annexing 0 to the right.

In like manner, it may be shown that to multiply by an exact number of hundreds, thousands, &c., we may first multiply by the figure denoting the hundreds, &c. as if it denoted ones, and then annex to the product two ciphers for hundreds, three for thousands, &c. If ciphers occur on the right of the multiplicand, they may be considered as transferred to the right of the multiplier. Thus 4600×30 is $46 \times 100 \times 30$, or 46×3000.

The form usually adopted for multiplication, when ciphers are on the right of the multiplier, is exhibited in the three following examples:—

2573	2573	2573000
40	4000	400
102920	10292000	1029200000

Examp. Multiply 478 by 16; and by 57.

These multipliers are beyond the limits of the Multiplication Table, but may easily be accommodated to it. For 16 times a number is the amount of 6 times + 10 times that number; and 57 times is the amount of 7 times + 50 times.

478	
16	
2868 = 6 times	3346 = 7 times
4780 = 10 times	23900 = 50 times
7648 = *16* times	27246 = 57 times

The pupil is now required to perform the second set of Exercises, Nos. 21 to 36; and for most of these he should employ two methods, viz. that which we have just exemplified, and that which resolves the multiplier into two factors. Thus, 16 being twice 8 times, if we had first multiplied 478 by 8, and then the product by 2, we should have obtained the same result as above.

The latter method, when the multiplier is a product of two factors, neither exceeding 12, will serve as a means of *proof*, but is not to be regarded as being always the preferable mode of working for Simple Multiplication. It has a superior expediency in Compound Multiplication.

The third set of Exercises does not conveniently admit the latter of the above methods, as no one of the multipliers is the product of two factors occurring in the Table. The pupil must now, therefore, give chief attention to the former method, which is general in its application, and is usually simplified by the omission of those right-hand ciphers that are uniformly occasioned by the tens, hundreds, &c. of the multiplier. We subjoin three examples, excluding the ciphers from the last, and connecting it with a mode of proof which we shall presently explain.

46957	36859	5864...	5
468	64005	2054...	2
375656	184295	23456	10
2817420	147436000	29320	
18782800	2211540000	11728	
21975876	2359160295	12044656...	1

In these examples we find the entire product by the addition of three partial products. By the first operation we add 8 times + 60 times + 400 times the multiplicand; by the second we add 5 times + 4000 times + 60000 times the multiplicand; by the third

we add 4 times + 50 times + 2000 times the multiplicand.

RULE.—Place the multiplier under the multiplicand, and, proceeding from right to left, multiply each figure of the multiplicand, first, by the simple units of the multiplier, next, by the tens, if any, and so forth, setting down and carrying as in Addition, and observing to place the right hand figure of each partial product under its own multiplier. Lastly, add the partial products into one sum.

A method of *proof* called *casting out the nines*, is often applied to Multiplication. Thus, for the third of the above examples, add the figures of the multiplicand, as if each represented simple units; drop *nine* as often as it arises in the summing, and place the result a little to the right:—According to this process, we say 5 and 8 gives 4 (*i. e.* 13 dropping 9), and 6 gives 1, and 4 gives 5. Place 5 on the right. Apply the same process to the multiplier and the product; the results will be 2 and 1. Lastly, multiply together the two uppermost results, which gives 10; and, as casting out the 9 from this leaves 1, which has already resulted from the main product, the exercise is presumed to have been correctly performed.

Casting out the *nines* is only a *presumptive* test of accuracy, as errors may have been committed which it cannot detect. It is, however, generally to be depended on, and is recommended by its simplicity.

Any number, used as a *Divisor* of the multiplicand, multiplier, and product, would furnish a presumptive test; but as the pupil is not yet considered capable of performing Division, the number 9 is chosen as one which leaves the same remainder, after dividing a number, as it would leave after dividing the sum of the digits of that number. The number 3 also has this peculiarity.

200 ÷ 9 leaves 2
80 ÷ 9 leaves 8
4 ÷ 9 leaves 4
284 ÷ 9 leaves 5

EXERCISES.

1. 57362483	×	2	11. 4629607	×	7
2. 19476392	×	2	12. 549875340	×	7
3. 59248360	×	3	13. 75610098	×	8
4. 27199016	×	3	14. 432690438	×	8
5. 39674852	×	4	15. 358617392	×	9
6. 23179540	×	4	16. 6402739	×	9
7. 4236918	×	5	17. 36285947	×	11
8. 17873565	×	5	18. 4790378	×	11
9. 93748562	×	6	19. 560972584	×	12
10. 13919540	×	6	20. 2899364	×	12

21. 6395384	×	14	29. 1916377	×	56
22. 3487608	×	15	30. 788940	×	80
23. 2493657	×	21	31. 209637	×	900
24. 7391628	×	10	32. 65323140	×	84
25. 7853926	×	20	33. 8469000	×	120
26. 1081923	×	60	34. 474389	×	108
27. 4362075	×	27	35. 532874	×	132
28. 2360490	×	35	36. 128269	×	144

Answers. 1. 114724966; 2. 38952784; 3. 177745080;
4. 81597048; 5. 158699408; 6. 92718160;
7. 21184590; 8. 89367825; 9. 562491372;
10. 83517240; 11. 32407249; 12. 3849127380;
13. 604880784; 14. 3461523504; 15. 3227556528;
16. 57624651; 17. 399145417; 18. 52694158;
19. 6731671008; 20. 34792368; 21. 89535376;
22. 52314120; 23. 52366797; 24. 73916280;
25. 157078520; 26. 64915380; 27. 117776025;
28. 82617150; 29. 107317112; 30. 63115200;
31. 188673300; 32. 5487143760; 33. 1016280000;
34. 51234012; 35. 70339368; 36. 18470736;

37.	5237468	× 17	53.	3581647	× 760
38.	3592406	× 19	54.	5466280	× 856
39.	26573859	× 23	55.	3856900	× 970
40.	2418276	× 26	56.	867074	× 1728
41.	73560483	× 39	57.	293567	× 2436
42.	1423895	× 53	58.	799389	× 3249
43.	5008637	× 67	59.	586000	× 4800
44.	80109633	× 78	60.	716178	× 5367
45.	68249753	× 94	61.	393045	× 7536
46.	9795241	× 167	62.	796528	× 6009
47.	3937594	× 230	63.	189736	× 7854
48.	7391089	× 324	64.	682174	× 6298
49.	2369570	× 365	65.	274389	× 7392
50.	9363125	× 379	66.	648557	× 8045
51.	8596384	× 437	67.	586470	× 9612
52.	4793530	× 508	68.	28934	× 98117

69. How many oranges are there in 28 boxes, each box containing 268? and how many sound ones are there, if half-a-dozen in each box are spoiled?

70. What number is 8 times the product of 2537 × 54?

71. If 10000 be lessened by the continued subtraction of 369; what remains after *seventeen* subtractions?

72. Multiply 108397 by 24856.

Answers.

37. 89036956; 38. 68255714; 39. 611198757; 40. 62875176; 41. 2868858837; 42. 75466435; 43. 335578679; 44. 6248551374; 45. 6415476782; 46. 1635805247; 47. 905646620; 48. 2394712836; 49. 864893050; 50. 3548624375; 51. 3756619808; 52. 2435113240; 53. 2722051720; 54. 4679135680; 55. 3741193000; 56. 1498303872; 57. 715129212; 58. 2597214861; 59. 2812800000; 60. 3843727326; 61. 2961987120; 62. 4786336752; 63. 1490186544; 64. 4296331852; 65. 2028283488; 66. 5217641065; 67. 5637149640; 68. 2838917278; 69. 7504; 7336; 70. 1095984; 71. 3727; 72. 2694315832;

73. Add together 47, 738, 36, 59, and 736; multiply the sum by 37; and tell how much must be added to the product to make *one million.*

74. Required the product of 596238 × 391 × 37.

V.—SIMPLE DIVISION.

Exemplification:—Suppose that with 18 shillings I have bought some articles which cost 3 shillings a-piece; the number of articles is 6; because 3 shillings are contained in 18 shillings 6 times.

Or, suppose I have 18 shillings to divide equally among 6 persons, and I am to consider what each person should receive; I ask what number of shillings is contained 6 times in 18 shillings, or what is the 6th part of 18 shillings; and I find the answer to be 3 shillings.

Definition.—DIVISION finds how often one number contains another, or, what number is contained a given number of times in another.

As Multiplication is a short method of doing a particular species of Addition, so Division is a short method of doing a particular species of Subtraction.

The first example in the margin supposes a person, originally possessing *no* pence, to receive three several sums of 8 pence; the second supposes a person originally possessing 24 pence to give away 8 pence as many times as his money permits. Such examples may be employed to illustrate the relation of Multiplication to Division, and their

0	24
8	8 *one* time
—	—
8	16
8	8 *two* times
—	—
16	8
8	8 *three* times
—	—
24	0

Answers. 73. 940208; 74. 8625775146;

respective relations to Addition and Subtraction. By the first process it is found that 3 times 8 make 24; by the second, that 24 contains 8 exactly 3 times.

As Multiplication proposes two factors, and requires their product, Division proposes the product and either factor, to find the other factor; thus 30 divided by 6 gives 5, or divided by 5 gives 6. For illustration of this interchange of divisor and quotient, reference may be made to the diagram given under Multiplication.

It is by having learned the Multiplication Table that we are enabled to perform Division. But though a thorough acquaintance with that Table implies a ready power of naming the factors which make any of its products, the pupil, when entering upon the study of Division, should be practised by such questions as these:—5 times what number make 30? How often is 6 contained in 48? How often is 7 contained in 34? Is 19 exactly divisible by 3? Name a number exactly divisible by 3; &c.

The name *dividend* is employed to denote the number to be divided; the *divisor* is the number by which we divide; and the *quotient* is the number of times which the dividend contains the divisor.

The sign of Division is $\div$; thus, $56 \div 8 = 7$. Frequently, the upper and under dots of the sign are replaced by the dividend and divisor, respectively; so that we may write $\frac{56}{8} = 7$. Both expressions signify that 56 *divided by* 8 *equals* 7.

As in Multiplication the proper multiplier is always *abstract*, so in Division the proper quotient is always *abstract*. Also, the dividend and divisor, like the multiplicand and product, are always of one name. Therefore, in such an expression as 12 yards $\div$ 3 = 4 yards, we should account 3 as being properly the quotient, *i. e.* the number which indicates *how often* the required quantity shall be contained in 12 yards.

The division of large numbers is conveniently performed by using a succession of partial dividends. We shall exemplify first with divisors not exceeding 12.

Divide 9774 by 3; and 23038 by 5.

Write the divisor on the left of the dividend, and draw lines of separation

3)9774
3258

as in the margin. Then 3 in 9 thousand goes 3 thousand times; 3 in 7 hundred goes 2 hundred times, leaving 1 hundred over, which, as being equal to 10 tens, makes the next partial dividend 17 tens; 3 in 17 tens goes 5 tens of times, leaving 2 tens over or 20 ones, making the next partial dividend 24 ones; 3 in 24 ones goes 8 times. We find, then, that 3 is contained in 9774 exactly 3258 times.

In the second example the first partial dividend to be taken is 23 thousand, as 2 by itself is not a sufficient number of tens of thousands to yield a quotient figure of that higher denomination. At the end of the division we find 3 simple units remaining over to be divided by 5; now as 5 is contained in 5 the 5th part of 5 times, therefore, 5 is contained in 3 the 5th part of 3 times, or $\frac{3}{5}$ of a time; this is called a fraction (*i. e.* part of a *broken* quantity), and is read 3 *fifths*. Accordingly 5 is contained in the proposed dividend 4607 times and 3 fifth parts of a time.

5)23038

$4607\frac{3}{5}$

When the pupil has wrought the first twenty Exercises, and has, occasionally at least, proved his work, by multiplying the quotient by the divisor, and adding to the product whatever remainder may have been left undivided, he may proceed to the consideration of Composite Divisors, *i. e.* Divisors composed of two factors each greater than unity.

When a divisor is a composite number, we may divide successively by its factors. When, for example, 27 is the divisor, we may divide first by 3 and then by 9.

If we mark 27 counters, it will be evident that, as the third part of them is 9, and the 9th part of 9 is 1, therefore the 27th part of them (viz. 1) is equal to the 9th part of the 3rd part of them. Or thus: When we have found how many divisions of 3 the dividend contains, we can argue that the divisions of 27 are one 9th part as many as the divisions of 3.

Let it be required to find what is the *twenty-eighth*

part of 10220; also how many *twenty-sevens* there are in 68769.

7)10220	3)68769
4)1460 = the 7th of 10220	9)22923 *threes* in 68769
365 = the 4th of the 7th	2547 *twenty-sevens*

Suppose, next, that we are required to find how many *thirty-sixes* are in 13894.

4)13894 *ones.*	2 ones + 8 fours = 34 ones, remainder.
9)3473 *fours* + 2 *ones*	
385 *thirty-sixes* + 8 fours	*Ans.* $385\frac{34}{36}$ *thirty-sixes.*

The above example will evince the reason of the ordinary Rule for collecting the entire remainder from partial remainders; the Rule being, to multiply the first divisor by the second remainder, and add to the product the first remainder. The extension of that Rule to suit an example like the following will be easily understood.

Find the quotient of $18212 \div 216$.

The divisor is equal to $4 \times 6 \times 9$; therefore,

4)18212 *ones*

6)4553 *fours*

9)758 *twenty-fours* + 5 *fours*

84 *two hundred and sixteens* + 2 *twenty-fours.*

The full remainder consists of 5 fours + 2 twenty-fours = 20 + 48 = 68.

In the following example it will be seen that, to divide by 10, we need only to consider the dividend, without its right-hand figure, as the quotient, and the right-hand figure as the remainder. Similar simplicity pertains to the divisors 100, 1000, &c.

How often is eighty contained in 47547?

10)47547

8)4754 *tens* + 7 *ones*

594 *eighties* + 2 *tens*

7 ones + 2 tens = 27 ones, remainder.

Ans. $594\frac{27}{80}$ times.

To save figures, the above example may be wrought as in the margin, by cutting off the right-hand figures of divisor and dividend, and dividing at once by 8.

8,0)4754,7

$594\frac{27}{80}$

These examples and explanations will prepare the pupil for performing Exercises 21 to 36; after which he will enter upon the most general method of Division, commonly called Long Division.

The operations we have already exemplified are those of Short Division. We shall illustrate the method of Long Division by an example which can be worked by Short Division.

Required the quotient of 28760 ÷ 8.

Here we see first, that 2 tens of thousands, divided by 8, gives no tens of thousands for the quotient. Accordingly, we must take 28 thousand as the first partial dividend suiting the divisor; and we find that the highest number of thousands which division by 8 will yield is 3; therefore as 3 thousand times 8 is 24000, there remains yet of the dividend, 4760; of this we take 47 hundred as the second partial dividend suiting the division; and the highest number of hundreds which it gives when divided by 8 is 5; then as 5 hundred times 8 is 4000, the remainder of the dividend is 760; of this we take 76 tens, yielding for the quotient 9 tens; and so forth, till the entire dividend is exhausted, and the entire quotient is found to be 3595.

8)28760(3000		
24000		
4760	500	
4000		
760	90	
720		
40	5	
40	—	
—	3595	

It is manifest, however, that such a mode of division

is necessary only when the divisor employed is a number too great to allow the successive multiplications and subtractions to be performed with ease, mentally.

It is also manifest, that the places which uniformly belong to ciphers may be left blank; and hence the subjoined specimens exhibit the most convenient methods of performing long division. The second form is preferred by the author as assigning to the quotient a position more natural, and more conducive to accuracy in bringing down the successive figures of the dividend.

Divide 1823117 by 479.

479)1823117(3806$\frac{43}{479}$	479)1823117
1437	———
———	3806$\frac{43}{479}$
3861	1437
3832	———
———	3861
2917	3832
2874	———
———	2917
43	&c.

RULE.—Write the divisor on the left of the dividend with a curve between them, and a line under the dividend. Find how often the divisor is contained in the first figure, or in the first sufficient number of figures, of the dividend, and place the number of times as the first quotient figure under the last figure of the partial dividend; from this dividend subtract the product of the divisor by the quotient figure, and to the remainder annex the next figure of the dividend for a new partial dividend; find how often this dividend contains the divisor, and write the number of times as the next quotient figure. Proceed similarly till all the figures of the given dividend have been used, and write the final remainder, if any, over the divisor, with a line between them, as the last portion of the quotient.

EXERCISES.

1. 65318944	÷	2	11. 330068886	÷	7
2. 95907368	÷	2	12. 946821692	÷	7
3. 75821841	÷	3	13. 3782921	÷	8
4. 79474614	÷	3	14. 7798904	÷	8
5. 86789536	÷	4	15. 39308335	÷	9
6. 18617419	÷	4	16. 52528253	÷	9
7. 487531900	÷	5	17. 6931045	÷	11
8. 818702633	÷	5	18. 51538931	÷	11
9. 221112240	÷	6	19. 821964900	÷	12
10. 809521429	÷	6	20. 88783543	÷	12
21. 10494096	÷	24	29. 6711579	÷	81
22. 26259128	÷	28	30. 34275355	÷	84
23. 1427970	÷	30	31. 9329360	÷	90
24. 17896066	÷	49	32. 3201146	÷	700
25. 2309783	÷	60	33. 1057188	÷	132
26. 30488059	÷	63	34. 681540	÷	800
27. 37150656	÷	64	35. 932572	÷	135
28. 6111338	÷	77	36. 6767154	÷	147

Answers. 1. 32659472; 2. 47953684; 3. 25273947;
4. 26491538 5. 21697384;
6. 4654354 *rem.* 3; 7. 97506380;
8. 163740526 *rem.* 3; 9. 36852040;
10. 134920238 *rem.* 1; 11. 47152698;
12. 135260241 *rem.* 5; 13. 472865 *rem.* 1;
14. 974863; 15. 4367592 *rem.* 7;
16. 5836472 *rem.* 5; 17. 630095;
18. 4685357 *rem.* 4; 19. 68497075;
20. 7398628 *rem.* 7; 21. 437254; 22. 937826;
23. 47599; 24. 365225 *rem.* 41;
25. 38496 *rem.* 23; 26. 483937 *rem.* 28;
27. 580479; 28. 79368 *rem.* 2;
29. 82859; 30. 408039 *rem.* 79;
31. 103659 *rem.* 50; 32. 4573 *rem.* 46;
33. 8009; 34. 851 *rem.* 740;
35. 6907 *rem.* 127; 36. 46035 *rem.* 9;

37.	48862205	÷ 19	47.	3724021	÷ 497
38.	9971783	÷ 38	48.	3975974	÷ 580
39.	45122679	÷ 47	49.	18686367	÷ 603
40.	72505332	÷ 53	50.	7831982	÷ 2648
41.	6219649	÷ 76	51.	1671436	÷ 3095
42.	12486813	÷ 85	52.	12605670	÷ 7854
43.	7228382	÷ 98	53.	10465686	÷ 8306
44.	7232017	÷ 149	54.	1472005	÷ 9200
45.	1532756	÷ 274	55.	920500	÷ 36820
46.	3535025	÷ 365	56.	1336225	÷ 45967

57. If 20280 trees are arranged in 78 equal rows, how many trees are in a row?

58. What number is 9 times the quotient of 792150 ÷ 25?

59. What number is contained exactly 39 times in 2325843?

60. Divide 500 times 99383 by 23 times 596.

61. What is the 5th part of thrice the 4th part of 1440700?

62. Add 46 + 239 + 86 + 425; multiply the sum by half of itself; subtract 1225 from the product; and divide the remainder by 69.

Answers, 37. 2571695; 38. 262415 *rem.* 13;
39. 960057; 40. 1368025 *rem.* 7;
41. 81837 *rem.* 37; 42. 146903 *rem.* 58;
43. 73759; 44. 48537 *rem.* 4;
45. 5594; 46. 9685;
47. 7493; 48. 6855 *rem.* 74;
49. 30989; 50. 2957 *rem.* 1846;
51. 540 *rem.* 136; 52. 1605;
53. 1260 *rem.* 126; 54. 160 *rem.* 5;
55. 25; 56. 29 *rem.* 3182;
57. 260; 58. 285174;
59. 59637; 60. 3625;
61. 216105; 62 4573 rem. 46;

ARTIFICES FOR ABBREVIATING SIMPLE MULTIPLICATION.

In many cases of Simple Multiplication the factors are such that we can easily obtain the entire product, without finding so many partial products as there are digits in the multiplier.

1. The multipliers 5, 25, and 125, are, respectively, the half of 10, the fourth of 100, and the eighth of 1000; therefore, conceive one cipher to be annexed to the multiplicand, and take half, for 5 times; conceive two ciphers to be annexed, and take a fourth, for 25 times, &c.*

2. The multipliers 99, 999, &c., are respectively, one less than 100, 1000, &c.; therefore annex two, three, &c. ciphers to the given multiplicand, and then subtrac that multiplicand.

Examp. Multiply 483 by 25, and by 99.

4)483	48300	= 100 times
	483	= 1 time
Ans. 12075		
	Ans. 47817	= 99 times.

3. A multiplier can sometimes be resolved into aggregate parts, whose significant figures are so conveniently proportioned to each other, that the product by one aggregate part whose significant portion exceeds 12, may be readily obtained from the product by another whose significant portion does not exceed 12.

Examp. Multiply 5974 by 273, and by 756.

5974	5974
273	756
17922	41818
161298	334544
1630902	4516344

* Division also may be abbreviated, when 5, 25, or 125, is the divisor; for we may divide twice, four times, &c. the given dividend by 10, 100, &c. respectively.

In the former example, we first compute for 3 times, and secondly, for 27 times, which is 3 times × 9; the product from 3, therefore, is multiplied by 9, to obtain 27 times the given multiplicand.

In the latter example, we first compute for 7 times, and secondly, for 56 times, which is 7 times × 8; the product from 7, therefore, is multiplied by 8. It is always allowable to take the partial products in any order of succession, provided we are careful that the right-hand figure of each product stand under the right-hand figure of the multiplier from which it arises, or stand as many places from its multiplier as the other products from their multipliers.

Examp. Multiply 9612 by 987, and 86957 by 32849.

987	86957
9612	32849
11844	695656
94752	2782624
9487044	4173936
	2856450493

In the former example, we first find the product by 12, and then take 8 times that product, because 8 times 12 make 96.

In the latter example, we first compute for 8 times; then for 32 times, viz. 8 times × 4; then for 48 times, viz. 8 times × 6; but in adding the partial products, we include *once* the multiplicand to complete 49 times.

Examp. Multiply 49378 by 7854, and 5973 by 4635.

49378	5973
7854	4635
296268	17919
888804	89595
2666412	806355
387814812	27684855

The former example shows an abbreviated method of using a multiplier, which is frequently required in Practical Mathematics; the multiplier 5236, which is another constant number often used as a multiplier, allows the very same kind of simplification. The process of multiplying by 7854 consists in resolving that multiplier into 6000 + 1800 + 54, in order that we may multiply first by 6, then the result by 3, and then the last result by 3; for 18 times = 6 times × 3; and 54 times = 18 times × 3. In the latter example, the multiplier 4635 is resolved into 3000 + 1500 + 135, so that we compute for 3, 5 times 3, and 9 times 15.

Examp. Multiply 7385 by 365; also 9746 by 5286, and 836 by 892.

7385	9746	836
365	5286	892
36925	58476	6688
258475	467808	76912
2695525	467808	745712
	51517356	

In the first example, we first obtain the product by 5, *i. e.* divide ten times the multiplicand by 2 and place the figures as they arise in division; then we find the product by 35, which is just 7 times the line already found; lastly, to complete 36 times, we include the multiplicand in the addition.

In the second example, we resolve the multiplier into 6 + 480 + 4800; consequently, we first take 6 times, then 8 times the result, then a repetition of 8 times.

In the third example, let the product by 8 stand under the multiplicand, as if two ciphers were annexed to the latter; subtract the product by 8 from 83600, *i. e.* from the product by 100, and the remainder, extended two places to the right, will be the product by *92; because* 100 — 8 = 92.

It may perhaps be considered that some of the above modes of abbreviation are not such as would readily suggest themselves; and that therefore our exemplification of them is of no great practical utility: but we are sure that a little practice in trying to discover such contractions will induce a quickness of perception, enabling the pupil to perform his multiplications in very many instances with greater expedition and convenience than the ordinary method allows.

VI. REDUCTION.

Numbers, in their application to the business of life, obtain names and values in addition to that of units. The number 236, for example, has its abstract or general value determined by the Numeration Table. But, while it always denotes 236 *units*, it may become a *concrete* number, by assuming an additional name, such as *inches*, *acres*, &c. Thus, 236 *inches* denotes 236 units of measure, each an inch long.

Now a single unit of one denomination may be made equivalent to several units of an inferior denomination. Thus, 1 *mile* consists of 8 equal parts, called *furlongs*; 1 *shilling* is divided into 12 *pence*; 1 *pound* weight is equivalent to 16 *ounces*; &c.

In the Manual, as well as in the present volume, will be found Tables of such equivalents; and the pupil may be required to commit these Tables to memory, one by one, as preparation for the exercises in Reduction, &c., may render necessary.

Definition. REDUCTION is the process of converting a number of one denomination into a corresponding number of another denomination.

Examp. Let it be required to change 280 shillings into pence, and also into sovereigns.

As 1 shill. equals 12 pence, 280 shill. equal 280 times 12 pence; therefore, we multiply 12 by 280, or, for greater convenience, we multiply 280 by 12, and obtain 3360 pence as equivalent to 280 shill.

280 times
12 pence

3360 pence

Secondly, take 20 shill. out of 280 shill. as often as can be done, because every 20 shill. makes 1 sovereign. The division gives 14 times 20 shill., *i. e.* 14 sovereigns.

20) 280 shill.
14 sov.

Examp. Reduce 14160 farthings to crowns.

There are so many pence in the given sum as the number of times it contains 4 farthings; also, there are so many shill. in the sum as its equivalent number of pence contains 12 pence; lastly, there are so many crowns in the sum as its equivalent number of shill. contains 5 shill. Thus, by gradually lessening the numbers, and raising the denominations from farthings to crowns, we obtain 59 crowns as the answer. The accuracy of the process nay be *proved* by reducing 59 crowns to farthings.

4) 14160 farth.
12) 3540 pence
5) 295 shill.
59 crs.

These explanations prepare the way for working the first fourteen Exercises. Let the pupil be occasionally required to test the correctness of his result, by reversing the problem.

Concrete quantities are of two kinds, Simple and Compound. A simple concrete quantity consists of only one denomination; as 23 miles. A compound concrete quantity consists of more than one denomination of the same kind; as 23 miles 6 furlongs.

We shall now exemplify Reduction involving compound quantities.

Examp. Reduce £8. 13*s*. 6*d*. to pence.

We employ the usual abbreviations £. *s*. *d*. to denote pounds, shillings, and pence, respectively. Now £8. are 8 times 20*s*. = 160*s*., and 13*s*. added make 173*s*. Again, 173*s*. are 173 times 12*d*. = 2076*d*., and 6*d*. added, make 2082*d*.

£8. 13*s*. 6*d*.
20
173*s*.
12
2082*d*. *Ans*.

Examp. Reduce 26794 farthings to £.

We first inquire how often 4 farthings is contained in the given quantity; the number of times is the equivalent number of pence. We next inquire how often 12 pence is found in the sum of pence, and thus we obtain, as an equivalent, a number of shillings with $2\frac{1}{4}d.$ over. Lastly, we inquire how often 20*s.* is found in the sum of shillings; and to the pounds and odd shillings thus obtained we annex the reserved pence.

4)26794*q.*
12)6698$\frac{1}{4}d.$
2,0)55,8*s.* $2\frac{1}{4}d.$
Ans. £27. 18*s.* $2\frac{1}{4}d.$

The above examples make it observable that a higher denomination is reduced to a lower by multiplication; and a lower to a higher by division.

When the pupil has arrived at the 79th Exercise he will require to be informed that in some cases Reduction is *indirect*, and involves both multiplication and division, as in the following example.

Examp. Convert 436 English ells into yards.

As there is not an exact number of yards in an English ell, we first reduce the given quantity to the most convenient denomination of which a yard and an ell severally contain an exact number; that is, we first change the English ells to quarters; and then we can directly convert the quarters into yards.

436 E. e.
5
4)2180 qrs.
Ans. 545 yds.

Otherwise, we might have found the number of yards in 436 *quarters*, that is $436 \div 4 = 109$ yds.; and then it would have remained to multiply 109 by 5, for there must be 5 times as many yards in 436 *E. e.* as in 436 *qrs.* It is, however, generally preferable to make division the concluding process, on account of the remainders it may yield.

EXERCISES.

1. Reduce 392 pence to farth. *Ans.* 1568*f.*
2. ..... 47 shill. to pence. *Ans.* 564*d.*

3. Reduce 806 shill. to farth. *Ans.* 38688*f.*
4. ..... £59 to pence. *Ans.* 14160*d.*
5. ..... £590 to shill. *Ans.* 11800*s.*
6. ..... £43 to hf. pence. *Ans.* 20640 *hf. d.*
7. ..... 73 guineas to pence. *Ans.* 18396*d.*
8. ..... 347 crowns to farth. *Ans.* 83280*f.*
9. ..... 1944 farth. to pence. *Ans.* 486*d.*
10. ..... 94680 shill. to £. *Ans.* £4734.
11. ..... 30000 farth. to shill. *Ans.* 625*s.*
12. ..... 23280 pence to £. *Ans.* £97.
13. ..... 6720 farth. to £. *Ans.* £7.
14. ..... 9576 hf. pence to guin. *Ans.* 19*g.*
15. ..... £37. 18*s.* to shill. *Ans.* 758*s.*
16. ..... £148. 10*s.* 4*d.* to pence. *Ans.* 35644*d.*
17. ..... £56. 12*s.* to pence. *Ans.* 13584*d.*
18. ..... £18. 8*s.* 7*d.* to farth. *Ans.* 17692*f.*
19. ..... £9. 0*s.* 11*d.* to hf. pence. *Ans.* 4342*hf. d.*
20. ..... 17*s.* 8*d.* to farth. *Ans.* 848*f.*
21. ..... £63. 16*s.* 5½*d.* to farth. *Ans.* 61269*f.*
22. ..... £6784. 9*s.* 10½*d.* to farth. *Ans.* 6513114*f.*
23. ..... £10. 15*s.* 0¾*d.* to farth. *Ans.* 10323*f.*
24. ..... 15*s.* 9½*d.* to hf. pence. *Ans.* 379 *hf. d.*
25. ..... 777 shill. to £. *Ans.* £38. 17*s.*
26. ..... 12854 pence to £. *Ans.* £53. 11*s.* 2*d.*
27. ..... 954 farth. to shill. *Ans.* 19*s.* 10½*d.*
28. ..... 96843 farth. to £. *Ans.* £100. 17*s.* 6¾*d.*
29. ..... 37273 farth. to guin. *Ans.* 36*g.* 20*s.* 6¼*d.*
30. ..... 563½ pence to £. *Ans.* £2. 6*s.* 11½*d.*
31. ..... 285730 farth. to £. *Ans.* £297. 12*s.* 8¼*d.*
32. ..... 10305 farth. to sovs. *Ans.* £10. 14*s.* 8¼*d.*
33. ..... 1657 guin. to sixpences. *Ans.* 69594*si.*
34. ..... £9. 16*s.* 10½*d.* to hf. pence. *Ans.* 4725*hf. d.*
35. ..... £136. 13*s.* to fourpences. *Ans.* 8199*fo.*
36. ..... £29. 14*s.* to hf. sovs. *Ans.* 59*hf. sov.* 4*s.*
37. ..... 38 sovs. 42 crs. 26*d.* to pence. *Ans.* 11666*d.*
38. ..... 826 crs. 4*s.* 8*d.* to fourpences. *Ans.* 12404*fo.*
39. ..... 929 fourpences to £. *Ans.* £15. 9*s.* 8*d.*
40. ..... 37 hf. crs. to threepences. *Ans.* 370*thr.*
41. ..... £57. 1*s.* 6*d.* to twopences. *Ans.* 6849*two.*

42. Reduce 1930 hf. sovereigns to hf. pence.
Ans. 463200*hf. d.*
43. ..... 23757 hf. pence to crowns.
Ans. 197 *crs.* 4*s.* 10½*d.*
44. ..... 1905 sixpences to £. *Ans.* £47. 12*s.* 6*d.*
45. ..... 12 cwt. to lbs. *Ans.* 1344*lbs.*
46. ..... 26 cwt. 2 qrs. 15 lbs. to lbs. *Ans.* 2983*lbs.*
47. ..... 79 cwt. 3 qrs. to lbs. *Ans.* 8932*lbs.*
48. ..... 47 cwt. 14 lbs. to lbs. *Ans.* 5278*lbs.*
49. ..... 8 cwt. 8 lbs. to oz. *Ans.* 14464*oz.*
50. ..... 3 qrs. 13 lbs. to lbs. *Ans.* 97*lbs.*
51. ..... 2367 lbs. to cwt. *Ans.* 21*cwt.* 15*lbs.*
52. ..... 33649 oz. to cwt.
Ans. 18*cwt.* 3*qrs.* 3*lbs.* 1*oz.*
53. ..... 5347 stones (of 14 lbs. each) to hf. oz.
Ans. 2395456*hf. oz.*
54. ..... 365 stones (of 8 lbs. each) to cwt.
Ans. 26*cwt.* 1*sto.*
55. ..... 46 tons 1 qr. 17 lbs. to hf. lbs.
Ans. 206170 *hf. lbs.*
56. ..... 12157 lbs. to tons. *Ans.* 5*t.* 8*cwt.* 2*qrs.* 5*lbs.*
57. ..... 15000 hf. oz. to stones of 14 lbs.
Ans. 33*st.* 6*lbs.* 12*oz.*
58. ..... 21 lbs. 3 ozs. Troy to dwts. *Ans.* 5100*dwts.*
59. ..... 102624 grs. to lbs. Troy.
Ans. 17*lbs.* 9*oz.* 16*dwts.*
60. ..... 11 oz. 4 dwts. 22 grs. to grs. *Ans.* 5398*grs.*
61. ..... 63372 grs. to lbs. Troy. *Ans.* 11*lbs.* 12*grs.*
62. ..... 25 yds. 2 ft. 7 in. to inches. *Ans.* 931*in.*
63. ..... 11080 in. to yds. *Ans.* 307*yds.* 2*ft.* 4*in.*
64. ..... 56 ft. 3 in. to hf. inches. *Ans.* 1350*hf. in.*
65. ..... 2609 poles to miles. *Ans.* 8*mi.* 1*fur.* 9*po.*
66. ..... 43 mi. 40 yds. to feet. *Ans.* 227160*ft.*
67. ..... 237694 hf. inches to miles.
Ans. 1*mi.* 1541*yds.* 11*in.*
68. ..... 1680806 yds. to lengths of 12 miles each.
Ans. 79*len.* 7*mi.* 6yds.
69. ..... 67 English ells 3 qrs. to in. Ans. 3042in.
70. ..... 13257½ nails to yards.
Ans. 828yds. 2qrs. 1½na.

71. Reduce 3 ac. 2 ro. 31 po. to sq. poles. *Ans.* 591*po.*
72. ..... 1479 sq. po. to acres. *Ans.* 9*ac.* 39*po.*
73. ..... 57 gall. to pints. *Ans.* 456*pi.*
74. ..... 23589 quarts to hhds. of 63 gallons.
Ans. 93*hhds.* 38*gal.* 1*qt.*
75. ..... 16 days 13 ho. 47 min. to minutes.
Ans. 23867 *min.*
76. ..... 18 ho. 32 min. to sec. *Ans.* 66720*sec.*
77. ..... 5236754 sec. to weeks.
Ans. 8*wks.* 4*da.* 14*ho.* 39*min.* 14*sec.*
78. ..... 294763 hrs. to years of 365 days.
Ans. 33*yrs.* 236*da.* 19*ho.*
79. ..... 5638 four-pences to six-pences.
Ans. 3758*si.* 1*fo.*
80. ..... £1659 to guineas *Ans.* 1580*guins.*
81. ..... 315 stones of 8 lbs. to stones of 14 lbs.
Ans. 180*st.*
82. ..... £2579. 2*s.* to guins. *Ans.* 2456*guins.* 6*s.*
83. ..... 2675 guins. to £. *Ans.* £2808. 15*s.*
84. ..... 2769 yds. to English ells.
Ans. 2215*E.e.* 1*qr.*
85. ..... 255 guins. 1 crown to £. *Ans.* £268.
86. ..... 837 quarter-inches to hf.-yds.
Ans. 11*hf.yds.* 11¼*in.*
87. Convert 2387 packages of 16 oz. each into another number of packages, half of which shall contain 12 oz. and the other half 10 oz. each.
Ans. 3472*p.*

VII. COMPOUND ADDITION.

Definition.—COMPOUND ADDITION is the addition of compound quantities; that is, quantities of like kind, but of different denominations.

The various denominations of which money, weight, and measure consist, are added, subtracted, &c. on the same principle as is employed with simple numbers. *This may be illustrated* by the two subjoined examples.

1. Add together 5239, 1476, 3587, and 2865.

2. Add together £52. 3*s*. 9*d*., £14. 7*s*. 6*d*., £35. 8*s*. 7*d*., and £28. 6*s*. 5*d*.

Hund.	*Tens.*	*Ones.*
52	3	9
14	7	6
35	8	7
28	6	5
131	6	7

Pounds.	*Shill.*	*Pence.*
52	3	9
14	7	6
35	8	7
28	6	5
130	6	3

The sum of the ones is 27 ones, which divided by 10 ones gives 2 tens and 7 ones; the 7 ones being reserved, and the 2 tens carried; &c.

The sum of the pence is 27 pence, which, divided by 12 pence, gives 2 shill. and 3 pence; the 3 pence being reserved, and the 2 shill. carried to the column of shillings, whose sum is in like manner divided by 20 to get £.; &c.

In the first example, the amount is 131 hundreds, 6 tens, and 7 ones, or 13167; in the second, the amount is 130 pounds, 6 shill. and 3 pence, or £130. 6*s*. 3*d*.

In the first example, the denominations are *uniformly* related, and *abstract;* in the second, the denominations are *variously* related, and *concrete*:—The denomination of hundreds is *ten* times the value of tens, and that of tens is *ten* times the value of simple units; whereas the denomination of pounds is *twenty* times the value of shillings, and that of shillings is *twelve* times the value of pence.

In the first of the two following examples the parts of a penny are added as farthings or fourths, thus: 2 farthings and 3 farthings make 5 farthings, and 1 farthing makes 6 farthings, = 1 penny and 2 farthings, or $1\frac{1}{2}$ *d*. We therefore write $\frac{1}{2}$*d*. under the column of farthings, and carry 1 penny to the column of pence; &c.

In the *second* example, containing quantities in Troy

weight, we first find the sum of the grains to be 55, which divided by 24 gives 2 dwts. and 7 grains: the sum of dwts., 31, divided by 20, gives 1 oz. and 11dwts.; the sum of oz., 16, divided by 12, gives 1 lb. and 4 oz.; and lastly the sum of lbs. is 27.

£	s.	d.
29	12	6¼
36	15	11¾
85	9	7½
£151	18	1½

lb.	oz.	dwt.	gr.
15	3	13	20
9	10	10	14
2	0	6	21
lb.27	4	11	7

EXERCISES.

1.	2.	3.	4.
£8 15	£64 12	£656 18 3	£718 15 11½
9 13	78 8	347 19 7¼	256 17 0¼
7 15	69 15	568 7 5	39 10 9
9 18	84 7	649 19 6½	763 6 10¾
10 18	76 14	289 8 8¾	93 19 10¾

5.	6.	7.	8.
9s. 6d.	17s. 6d.	£409 2 2¾	£249 6 8¼
4 10	15 9½	368 4 9½	698 13 8
7 5	9 7¼	757 0 10	570 9 6¼
6 11	16 3	249 3 6¼	460 0 9
9 8	5 8¼	875 1 11½	773 13 2¾
8 10	13 5	680 5 5½	457 16 4

Answers. 1. £46 19s.; 2. £373. 16s.;
3. £2512. 13s. 6½d.; 4. £1872. 10s. 6¼d.;
5. £2. 7s. 2d.; 6. £3. 18s. 3d.;
7. £3338. 18s. 9½d.; 8. £3210. 0s. 2¼d.;

9.			10.			11.		
£396	15	10¾	£550	13	7	£713	9	2½
36	15	6½	679	3	1	393	10	0½
658	8	7¼	754	16	10½	61	4	0½
789	17	11½	88	12	0	95	14	1¾
493	3	8	235	4	11¼	404	3	0¾
96	0	10½	60	5	9	580	17	3¾
98	7	6	365	12	0	80	12	0¼

12. A workman's earnings, in seven successive weeks, were, respectively, 17*s.* 6*d.*, 18*s.* 6*d.*, 18*s.* 10*d.*, 16*s.* 7*d.*, 14*s.*, 16*s.* 11*d.*, and 17*s.* 5*d.* Required the sum total.

13. Paid for tea 4*s.* 9*d.*, for coffee 3*s.* 10½*d.*, for sugar 3*s.* 1½*d.*, for candles 1*s.* 6*d.*, for bacon 2*s.* 9*d.* Required the amount.

14. A horse-dealer bought four horses, for £34. 10*s.*, £28. 8*s.*, 40 guineas, and 56 guineas, respectively; and he sold them again at a profit of 3 guineas on each of the first two, and £5. 10*s.* on each of the last. What amount did he receive?

15.			16.			17.		
£749	14	6	£505	11	11	£468	0	4¼
859	16	8¾	420	19	3½	579	0	9½
47	6	6½	643	14	7¼	865	10	10
10	10	0	869	17	9	986	17	3½
296	7	5½	786	18	8½	657	16	7½
868	7	9	957	16	6	309	19	10½
75	15	6	895	15	7¾	866	8	5¼
143	8	7	678	17	9	490	15	1½
629	5	2½	689	13	5	749	6	8½

Answers. 9. £2569. 10*s.* 0½*d.*;
10. £2734. 8*s.* 2¾*d.*; 11. £2329. 9*s.* 10*d.*;
12. £5. 19*s.* 9*d.*; 13. 16*s.*; 14. £181;
15. £3680. 12*s.* 3½*d.*; 16. £6449. 5*s.* 7*d.*;
17. £5973. 15*s.* 11¾*d.*;

18.

Cwt.	qr.	lb.
46	2	15
73	3	3
86	1	27
59	3	11
94	2	25

19.

Cwt.	qr.	lb.	oz.
35	3	10	14
69	2	14	12
87	1	18	10
68	2	22	8
54	3	17	13

20.

T.	cwt.	qr.	lb.	oz.
2	17	3	11	13
1	13	1	14	14
8	16	2	20	9
5	8	0	11	14
3	19	3	7	6

21.

Yd.	ft.	in.
32	2	10
17	1	8
46	2	7
30	2	10
68	1	11

22.

Yd.	qr.	na.
25	2	1
13	3	3
27	3	2
48	1	1
26	2	3

23.

Mi.	yd.	ft.	in.
26	1350	2	10
307	223	2	10
678	459	1	9
75	1400	2	11
84	96	0	8

24.

Ac.	ro.	po.
59	2	23
5	0	16
7	3	37
20	2	28
3	1	30

25.

Ac.	ro.	po.
48	3	15
61	1	24
43	2	39
32	3	14
29	3	26

26.

Da.	ho.	min.	sec.
237	14	33	25
365	5	48	50
589	10	27	26
468	21	56	49
808	18	34	32

27. One waggon-train removes from a coal-store 330 cwt.; another train takes 375 cwt.; and then 64 cwt. 3 qrs. are carted away for the use of three families. There are 54 tons 15½ cwt. still to be disposed of. What quantity was there at first?

Answers. 18. 361*cwt.* 1*qr.* 25*lb.*;
19. 316*cwt.* 2*qr.* 9*oz.*; 20. 22*t.* 15*cwt.* 3*qr.* 10*lb.* 8*oz.*
21. 196*yds.* 2*ft.* 10*in.*; 22. 142*yds.* 1*qr.* 2*na.*;
23. 1172*mi.* 11*yds.* 2*ft.*; 24. 96*ac.* 3*ro.* 14*po.*;
25. 216*ac.* 2*ro.* 38*po.*; 26. 2469*da.* 23*ho.* 21*min.* 2*sec.*;
27. 93*t.* 5*cwt.* 1*qr.*

28. I paid £137. 13*s*. 7*d*. for a quantity of goods. They cost me, besides, for packing, £1. 12*s*. 10½*d*., and for carriage, £2. 10*s*. 9½*d*. For what sum must I sell them to gain 50 guineas?

29. A clerk is entrusted with a sum of money, to pay some bills: he pays one bill of £49. 12*s*. 8*d*.; another of £63. 8*s*. 9*d*.; a third of £32. 1*s*. 3½*d*.; and a fourth of £25. 14*s*. 8*d*.; and he returns a balance consisting of 2 sovereigns, 3 half-sovereigns, a crown, 3 half-crowns, and 3 half-pence. What sum did he receive?

VIII. COMPOUND SUBTRACTION.

The observations which have been made respecting Compound Addition are so obviously suggestive of the general illustration appropriate to the Subtraction of compound quantities, that here we shall merely furnish an example of the latter operation.

Examp. Subtract £138. 16*s*. 4½*d*. from £235. 13*s*. 9¼*d*.

£235	13	9¼
138	16	4½
£ 96	17	4¾

Beginning at the place of farthings, we say 2 farth. from 1 farth. cannot be taken; therefore (on the principle employed in Simple Subtraction), we add 4 farth. to the minuend and 1 penny to the subtrahend, and say 2 farth. from 5 farth. leaves 3 farth., and 5 pence from 9 pence leaves 4 pence; then, in order to subtract 16 shill., we add 20 shill. to the minuend and 1 pound to the subtrahend, and say 16 shill. from 33 shill. leaves 17 shill., and £139 from £235 leaves £96.

In cases where the shill., pence, &c. of the minuend are fewer than those in the subtrahend, the subtraction may be effected more easily by adding to the upper quantity the complement of the lower, or what it wants

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Answers. 28. £194. 7*s*. 3*d*.; 29. £175.

to make it of the next higher denomination. Thus, instead of saying 16*s.* from 33*s.*, we may say the complement of 16*s.* is 4*s.*, which added to 13*s.* make 17*s.*

EXERCISES.

1.	2.	3.
£368 12 9	£760 14 6	£522 8 9
295 7 3	329 13 8	229 16 10

4.	5.	6.
£84 10 3½	£148 3 5¾	£600 15 1¾
64 14 5¼	57 9 0¼	60 18 6½

7.	8.	9.	10.
£52 6	17*s.* 5½*d.*	32*s.* 6*d.*	£217 17 0¼
23 8	6 10¾	15 7½	182 10 8½

11.	12.	13.
£1060 1 5¼	£372 18 9½	£820 4 0
162 1 11¾	360 19 9½	13 9 11¾

14.	15.	16.
£313 14 0¾	£569 0 11½	£291 3 7
308 16 10¾	79 0 11¾	290 17 0½

Answers. 1. £73. 5*s.* 6*d.*; 2. £431. 0*s.* 10*d.*;
3. £292. 11*s.* 11*d.*; 4. £19. 15*s.* 10¼*d.*;
5. £90. 14*s.* 5½*d.*; 6. £539. 16*s.* 7¼*d.*;
7. £28. 18*s.*; 8. 10*s.* 6¾*d.*;
9. 16*s.* 10½*d.*; 10. £35. 6*s.* 3¾*d.*;
11. £897. 19*s.* 5½*d.*; 12. £11. 19*s.*;
13. £806. 14*s.* 0¼*d.*; 14. £4. 17*s.* 2*d.*;
15. £489. 19*s.* 11¾*d.*; 16. 6*s.* 6½*d.*;

17. Subtract 10*s.* 7*d.* from £50.

18. How much does £4. 10*s.* exceed 3½*d.*?

19. From £18. 13*s.* 9*d.* deduct 18*s.* 6¾*d.*

20. Required the difference between 14*s.* 8½*d.* and £1. 14*s.* 6¼*d.*

21.

£517	12	6½
471	1	10

22.

£905	0	1½
5	19	5½

23.

£1000	0	0
10	10	10

24.

Ac.	ro.	po.
342	1	12
42	2	22

25.

Mi.	yd.	ft.	in.
13	24	0	9
12	200	1	0

26.

T.	cwt.	qr.	lb.
14	6	0	0
6	13	3	13

27.

Yd.	ft.	in.
410	1	3
121	1	6

28.

Cwt.	qr.	lb.
436	3	17
9	3	21

29.

Ac.	ro.	po.
140	2	17
100	3	26

30. A grocer had a hogshead of sugar weighing 11 cwt. 3 qrs. 10 lbs.; of which he sold 1 cwt. 2 qrs. 15 lbs. to A, 2 cwt. 3 qrs. 13 lbs. to B, 2 cwt. 26 lbs. to C, and 4 cwt. 3 qrs. to D. He had then on hand only 23 lbs. 10 oz. Required the loss by retail weighing.

Answers. 17. £49. 9*s.* 5*d.*;
18. £4. 9*s.* 8¾*d.*; 19. £17. 15*s.* 2¼*d.*;
20. 19*s.* 9¾*d.*; 21. £46. 10*s.* 8½*d.*;
22. £899. 0*s.* 8*d.*; 23. £989. 9*s.* 2*d.*;
24. 299*ac.* 2*ro.* 30*po.*; 25. 1583*yds.* 2*ft.* 9*in.*;
26. 7*t.* 12*cwt.* 15*lbs.*; 27. 288*yds.* 2*ft.* 9*in.*;
28. 426*cwt.* 3*qrs.* 24*lbs.*; 29. 39*ac.* 2*ro.* 31*po.*;
30. 16*lbs.* 6*oz.*;

31. A's memorandum book records that he took with him to market £18. 12*s.* 4½*d.*; disbursed for tolls 1*s.* 4*d.*, and for dinner 4*s.* 6½*d.*; sold a horse for 16 guineas, and nine bullocks for £94. 10*s.* 6*d.*; paid shoemaker's account £5. 13*s.* 4½*d.*, and tailor's ditto, £7. 17*s.* 6*d.*; received payment of an account for grain £53. 14*s.* The sum he has brought home is £169. 15*s.* 6*d.* How does this agree with his book?

Answer. 31. 7½*d. short.*

IX. COMPOUND MULTIPLICATION.

For this Rule no special illustration seems to be requisite, beyond one or two examples.

Examp. Multiply £95. 17*s.* 5¾*d.* by 3.

Beginning at the place of farthings, we say 3 times 3 farthings is 9 farthings, or 2 pence and 1 farth.; reserve ¼*d.* and carry 2*d.*; 3 times 5*d.* is 15*d.*, and 2*d.* carried makes 17*d.*, or 1*s.* and 5*d.*; reserve 5*d.* and carry 1*s.*; 3 times 17*s.* is 51*s.*, and 1*s.* carried makes 52*s.*, or £2 and 12*s.*; reserve 12*s.* and carry £2; 3 times £95 is £285, and £2 carried makes £287.

£95	17	5¾
		3
£287	12	5¼

The above is an example of the method applicable to the first 24 Exercises. Those from the 25th to the 47th inclusive, have composite multipliers, which may therefore be resolved into factors, as explained under the head of Simple Multiplication.

For the performance of the next set of Exercises, the usual procedure is to break up the multiplier into its thousands, hundreds, &c.; then to find the products of the given multiplicand by these portions separately; and lastly, to add the several products thus obtained. But many persons prefer to calculate the product by

the unresolved multiplier. We shall exemplify both these methods in finding the product of £23. 7*s*. 5*d*. by 365.

5	times	=	£23	7	5	× 5	=	£116	17	1
					10					
60	times	=	233	14	2	× 6	=	1402	5	0
					10					
300	times	=	2337	1	8	× 3	=	7011	5	0
365	times	. . . . .					=	£8530	7	1

Otherwise.

5*d*.	× 365	= 1825*d*.	=	152*s*. 1*d*.	=	£7	12	1
7*s*.	× 365		=	2555*s*.	=	127	15	0
£23	× 365				=	8395	0	0
					Ans.	£8530	7	1

The latter will generally be found the more convenient method. But perhaps the most expeditious mode of using the particular multiplier 365, in Compound Multiplication, is to resolve it into 5 × 12 × 6 + 5; for the first product may be added in while the last multiplication is being performed.

EXERCISES.

	£.	*s*.	*d*.				£.	*s*.	*d*.		
1.	163	7	4	×	2	7.	89	17	11½	×	8
2.	824	5	6	×	3	8.	512	7	10½	×	9
3.	250	13	7	×	4	9.	875	15	9½	×	10
4.	609	16	5	×	5	10.	96	11	5½	×	11
5.	67	9	8½	×	6	11.	749	16	7½	×	12
6.	438	10	6¾	×	7						

Answers. 1. £326. 14*s*. 8*d*.;
2. £2472. 16*s*. 6*d*.; 3. £1002. 14*s*. 4*d*.
4. £3049. 2*s*. 1*d*.; 5. £404. 18*s*. 1½*d*.
6. £3069. 13*s*. 11½*d*.; 7. £719. 3*s*. 6*d*.
8. £4611. 10*s*. 10½*d*.; 9. £8757. 18*s*. 1½*d*.;
10. £1062. 5*s*. 9¾*d*.; 11. £8997. 19*s*. 6*d*.;

	Cwt.	qr.	lb.				Mi.	yd.	ft.		
12.	126	3	27	×	3	15.	326	15	2	×	9
	Ac.	ro.	po.				Yd.	qr.	na.		
13.	79	2	13	×	5	16.	207	3	2	×	10
	Yd.	ft.	in.				Ho.	m.	sec.		
14.	237	2	10	×	7	17.	36	53	40	×	11

18. Calculate 5 lbs. of rice, at 3½*d.* per lb.
19. ..... 6 stones flour, at 2*s.* 11*d.* per st.
20. ..... 7 yds. velvet, at 16*s.* 6*d.* per yd.
21. ..... 8 cwt. sugar, at 52*s.* 6*d.* per cwt.
22. ..... 9 tons coal, at 25*s.* 10*d.* per ton.
23. ..... 10 gall. brandy, at £1. 11*s.* 8*d.* per gall.
24. ..... 12 pecks potatoes, at 7½*d.* per peck.

	£.	*s.*	*d.*				£.	*s.*	*d.*		
25.	229	16	5½	×	24	31.	326	18	9	×	64
26.	469	13	10½	×	28	32.	64	7	0	×	70
27.	85	0	7¼	×	35	33.	417	8	10½	×	81
28.	53	17	0½	×	44	34.	49	14	8¼	×	96
29.	184	11	2¾	×	48	35.	156	10	11¾	×	100
30.	7	19	11½	×	60	36.	790	15	0½	×	144

37. What is the extent of an estate which can be divided into 32 equal portions, each 32 ac. 1 ro. 36 po.?

Answers. 12. 380*cwt.* 3*qrs.* 25*lbs.*
13. 397*ac.* 3*ro.* 25*po.*; 14. 1665*yds.* 1*ft.* 10*in.*;
15. 2934*mi.* 141*yds.*; 16. 2078*yds.* 3*qrs.*;
17. 405*h.* 50*m.* 20*s.*; 18. 1*s.* 5½*d.*;
19. 17*s.* 6*d.*; 20. £5. 15*s.* 6*d.*;
21. £21.; 22. £11. 12*s.* 6*d.*;
23. £15. 16*s.* 8*d.*; 24. 7*s.* 6*d.*;
25. £5515. 15*s.*; 26. £13151. 7*s.* 11*d.*;
27. £2976. 1*s.* 1¾*d.*; 28. £2369. 9*s.* 10*d.*;
29. £8858. 19*s.*; 30. £479. 17*s.* 6*d.*;
31. £20924.; 32. £4504. 10*s.*;
33. £33812. 18*s.* 10½*d.*; 34. £4774. 10*s.*;
35. £15654. 17*s.* 11*d.*; 36. £113868. 6*s.*;
37. 1039*ac.* 32*po.*;

38. Required the amount of 50 cwt. of tobacco, at £7. 15*s.* 8*d.* per cwt.

39. What is the weight of 49 hhds. sugar, each 15 cwt. 3 qr. 17 lb.?

40. How much cloth is in 54 pieces, each 23 yd. 1 qr. 3 na.?

41. If 17 mi. 5 fur. 23 po. be travelled each day, for 25 days; how much is that altogether?

42. If 120 men are to receive each £59. 17*s.* 8¾*d.*; what sum is required for all?

43. Find the amount of 112 lbs. of tea, at 4*s.* 9½*d.* per lb.

44. .... 135 tons of iron, at £9. 16*s.* 7*d.* per ton.

45. .... 154 quarters wheat, at £2. 19*s.* 9*d.* per qr.

46. .... 250 sacks of flour, at £2. 13*s.* 5½*d.* per sk.

47. .... 125 ounces of gold, at £3. 17*s.* 10½*d.* per oz.

	£.	*s.*	*d.*				£.	*s.*	*d.*		
48.	537	12	3¾	×	13	56.	76	14	9½	×	131
49.	967	1	8½	×	19	57.	67	11	4	×	255
50.	70	15	6	×	29	58.	40	17	6¼	×	365
51.	352	17	10¼	×	43	59.	8	13	2½	×	507
52.	430	4	7	×	67	60.	15	9	10	×	690
53.	91	10	1½	×	79	61.	26	6	10¼	×	969
54.	168	3	11¾	×	86	62.	48	15	5¾	×	2749
55.	639	13	5	×	93	63.	60	18	11	×	5073

Answers. 38. £389. 3*s.* 4*d.*; 39. 779*cwt.* 21*lbs.*

40. 1265*yds.* 2*qrs.* 2*na.*; 41. 442*mi.* 3*fur.* 15*po.*;

42. £7186. 7*s.* 6*d.*; 43. £26. 16*s.* 8*d.*;

44. £1326. 18*s.* 9*d.*; 45. £460. 1*s.* 6*d.*;

46. £668. 4*s.* 7*d.*; 47. £486. 14*s.* 4½*d.*;

48. £6989. 0*s.* 0¾*d.*; 49. £18374. 12*s.* 5½*d.*;

50. £2052. 9*s.* 6*d.*; 51. £15174. 7*s.* 8¾*d.*;

52. £28825. 7*s.* 1*d.*; 53. £7228. 19*s.* 10½*d.*;

54. £14465. 2*s.*, 2½*d.*; 55. £59489. 7*s.* 9*d.*;

56. £10052. 17*s.* 8½*d.*; 57. £17229. 10*s.*;

58. £14919. 15*s.* 1½*d.*; 59. £4390. 16*s.* 7½*d.*;

60. £10689. 5*s.*; 61. £25526. 1*s.* 8½*d.*;

62. £134079. 12*s.* 2¾*d.*; 63. £309178. 4*s.* 3*d.*;

64. If a man's weekly wages be 32*s.* 10*d.*, how much more does he make in a year than another man who has 26*s.* 3*d.* a week?

65. What quantity of grain is reaped from 736 acres, the average produce being 4 qrs. 5 bu. 1 pk. per acre? and what will the grain weigh, at 1 cwt. 2 qrs. 24 lbs. per qr.?

66. Required the joint amount given for 17 pigs at £1. 15*s.* 6*d.*, and 31 score of sheep at £1. 6*s.* 7*d.* a head. Also, how much cheaper are two dozen sheep than a score of pigs?

67. A sale is made of 46 yards linen at 1*s.* 10½*d.*, 7 pieces calico, each 24 yards, at 5½*d.* a yard, 13 muslin dresses at 7*s.* 6½*d.*, and 58 yards flannel at 1*s.* 7½*d.* Required the amount.

Answers. 64. £17. 2*s.* 4*d.*;
65. 3427*qrs.* = 5874*cwt.* 3*qrs.* 12*lbs.*;
66. £854. 5*s.* 2*d.*, and £3. 12*s.*; 67. £17. 14*s.* 4*d.*

Bills of Parcels.

68. London, Feb. 25th, 1852.

Mr. James Dixon,

Bought of James Parker.

5 lbs. tea,	at	4*s.* 8*d.*	.....£
9 lbs. sugar,	at	6½*d.*	.....
7 lbs. do.	at	5*d.*	.....
10 lbs. do.	at	7½*d.*	.....
8 lbs. coffee,	at	1*s.* 8*d.*	.....

£2. 10*s.* 8½*d.*

69. London, June 3rd, 1850. Mr. Edward Hicks buys of Mr. Thomas Jones—7 lbs. of bacon at 7½*d.*; a cheese weighing 6 lbs. at 9½*d.*; 10 lbs. butter at 13*d.*; and 2 dozen eggs at 9*d.* Write out the bill.

Ans. £1. 1*s.* 5½*d.*

70. Edinburgh, Aug. 7th, 1849. Mrs. Arthur buys of Messrs. Syme and Preston—17 yds. black satin at 8*s.* 6*d.*; 7 yds. lining at 4½*d.*; 10 pair of stockings at 3*s.* 4½*d.*; 3 pair of gloves at 2*s.* 6*d.*; 1 yd. silk velvet at 11*s.* 9*d.*; and 3 yds. ribbon at 8¾*d.* Make out the bill. *Ans.* £10. 2*s.* 3¾*d.*

71. Dublin, Mar. 20th, 1851. Mr. Francis Wilson buys of Mr. George Drummond—25 yds. of linen at 2*s.*; 7 yds. do. at 2*s.* 6*d.*; 3 yds. cambric at 10*s.* 9*d.*; 20 yds. diaper at 1*s.* 4*d.*; 14 yds. calico at 5¾*d.*; 18 yds. do. at 6½*d.*; 10 yards muslin at 1*s.* 4½*d.*; 5 yds. do. at 1*s.* 9*d.* Make out the bill. *Ans.* £8. 5*s.* 4½*d.*

72. London, July 21st, 1848. Mr. John Evans buys of Mr. Frederick Fitch—15 lbs. of beef at 7½*d.*; a leg of mutton, wt. 8 lbs., at 8*d.*; two quarters of lamb, wts. 11 lbs. 7 oz. and 10 lbs. 9 oz., at 8½*d.* per lb.; 5 st. 13 lbs. of pork at 6½*d.* per lb.; and four fillets of veal, each 13 lbs. 4 oz., at 9½*d.* per lb. Make out the bill. *Ans.* £5. 16*s.* 1½*d.*

X. COMPOUND DIVISION.

The first 18 Exercises in Compound Division propose to find Quotients properly so called:—They require to determine *how often* one concrete quantity is contained in another. When the dividend does not contain the divisor an exact number of times, the proper and entire quotient cannot be demanded from pupils unacquainted with the principles of fractional arithmetic: and therefore, in such a case, an undivided portion of the dividend may be allowed to stand over as a remainder.

Examp. Find how often 2*ro.* 34*po.* is contained in 22*ac.* 2*ro.* 10*po.*

As an expedient preparation, we reduce dividend and divisor to one denomination, that of *poles*. Then we find that the divi-

22*ac.* 2*ro.* 10*po.*
4
2*ro.* 34*po.* 90
40 40
114*po.*)3610*po.*

Ans. 31 *times.* *Rem.* 76*po.*

dend contains the divisor 31 *times*, with a remainder of 76 poles, or 1ro. 36 po. For the proper completion of the quotient we require to determine how often 114po. is contained in 76po.; but some familiarity with fractions is necessary to understand that this part of the quotient is 2 *thirds* of a time, and that the proper answer is $31\frac{2}{3}$ times.

The remaining exercises propose to find what divisor is contained a given number of times in a given dividend; but the given number of times is usually called the divisor, and the answer, the quotient.

Examp. What sum of money is contained 9 times in £237. 10*s.* 7½*d.*?

The 9th part of £237, or the highest number of £. contained 9 times in £237, is £26, with £3 remaining; that remainder makes, with the 10*s.*, a dividend of 70*s.*, the 9th part of which is 7*s.*, with 7*s.* remaining; the next dividend is 7*s.* 7*d.*, or 91*d.*, the 9th part of which is 10*d.*, with 1*d.* remaining; the last dividend is 1½*d.*, or 6 farth., the 9th part of which is less than a farthing. Let, therefore, 1½*d.* stand over as an undivided remainder.

9)£237. 10*s.* 7½*d.*

Ans. £26. 7*s.* 10*d.* *Rem.* 1½*d.*

The answer would be determined by fractional operation to be £26. 7*s.* $10\frac{1}{6}$*d.*, or £26. 7*s.* $10\frac{2}{4}$*d.* $\frac{3}{4}$*q.*

From the 35th Exercise to the 45th, the divisors are composite, and may be resolved into factors.

Examp. Required the 84th part of 243*da.* 15*ho.*

As 84 equals 7 × 12, we first find the 7th part of the given quantity, and then the 12th part of the 7th. At the end of the former division there remains 4 sec. undivided; and at the end of the latter division there is

7)243*da.* 15*ho.*

12)34*da.* 19*ho.* 17*min.* 8*sec.* *Rem.* 4*sec.*

2*da.* 21*ho.* 36*min.* 25*sec.* *Rem.* 60*sec.*

a remainder of 8 sec. *for each 7th part of the whole*, that is, a remainder of 56 sec., which with 4 sec. previously remaining make 60 sec. or 1 min. as the entire remainder.

The remainder divided by 84 would make the answer terminate properly with 25$\frac{1}{7}$ sec. And observe, it is optional whether the above division should be carried out to seconds; if we had ended with the denomination of minutes, the answer would have been 2 da. 21 ho. 36 min. *Rem.* 36 min., or 2 da. 21 ho. 36$\frac{1}{7}$ min.

For the performance of the 46th and most of the subsequent Exercises, the process of Long Division is requisite.

Examp. Divide £1563. 13*s.* 5$\frac{1}{2}$*d.* into 347 equal parts.

347)£1563. 13*s.* 5$\frac{1}{2}$*d.*

Ans. Each part is £4. 10*s.* 1$\frac{1}{2}$*d.* *Rem.* 1*d.*

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The annexed operation proceeds on the same principle of successive division which we have previously exemplified; only, the remainders, reductions, &c. in this example, are not employed mentally, but noted down, to relieve the memory

£175 *rem.*
20
3513*s.*
347
43*s.* *rem.*
12
521*d.*
347
174*d.* *rem.*
4
698 *q.*
694
4 *q.* *rem.*

EXERCISES.

	£.	s.	d.		£.	s.	d.		Answers.
1.	213	19	11	÷	1	6	7	=	161 times.
2.	430	13	4	÷	5	1	4	=	85 .....
3.	81	8	9¾	÷	0	10	7¾	=	153 .....
4.	33	11	6	÷	0	2	1½	=	316 .....
5.	163	3	9¾	÷	0	3	6¼	=	927 .....
6.	45	13	5	÷	6	2	4¼	=	7 ti. *rem.* 56*s.* 11¼*d.*
7.	29	11	3	÷	1	9	6½	=	20 ti. *rem.* 5*d.*
8.	36	3	10½	÷	0	13	11	=	52 ti. *rem.* 2½*d.*
9.	37	16	0	÷	0	0	6¾	=	1344 times.
10.	0	12	4¾	÷	0	1	9¼	=	7 .....
11.	50	0	0	÷	3	12	6	=	13 ti. *rem.* 57*s.* 6*d.*
12.	8	11	4½	÷	1	0	6	=	8 ti. *rem.* 7*s.* 4½*d.*

13. How many lbs. of sugar at 9½*d.* will £3. 0*s.* 11½*d.* purchase? *Ans.* 77*lbs.*

14. What quantity of silk, at 3*s.* 5*d.* a yard, amounts to £15. 10*s.* 11*d.*? *Ans.* 91*yds.*

15. In what time will a weekly payment of £5. 10*s.* 7*d.* amount to £287. 10*s.* 4*d.*? *Ans.* 52*wks.*

16. How many ounces of silver at 5*s.* 2*d.* an ounce, will amount to £2790? *Ans.* 10800*oz.*

17. How many times does a wheel of 1 yd. 8 in. circumference revolve in passing over 1 mile?

Ans. 1440 *times.*

18. What part of 637 cwt. is 2 tons 5 cwt. 2 qrs.?

Ans. The 14*th.*

	£.	s.	d.				Answers.
19.	578	14	8½	÷	2	=	£289 7*s.* 4¼*d.*
20.	263	12	9	÷	3	=	87 17 7
21.	149	11	7	÷	4	=	37 7 10¾
22.	78	19	3¼	÷	5	=	15 15 10¼
23.	100	10	4½	÷	6	=	16 15 0¾
24.	977	5	5½	÷	7	=	139 12 2½

	£.	s.	d.				Answers.			
25.	256	7	0¾	÷	8	=	£32	0s.	10½d.	rem. 0¾d.
26.	16	16	11	÷	9	=	1	17	5	rem. 2d.
27.	309	7	8½	÷	10	=	30	18	9¼	
28.	260	0	0	÷	11	=	23	12	8½	rem. 2½d.
29.	5	11	10¾	÷	12	=	0	9	3¾	rem. 1¾d.

30. What weight is the 8th part of 17 hogsheads of tobacco each 5 cwt.? *Ans.* 10*cwt.* 2*qrs.* 14*lbs.*

31. What quantity is contained 7 times in 91 ac. 2 ro. 11 po.? *Ans.* 13*ac.* 13*po.*

32. Required the 9th part of 8 ac. 2 ro. 35 po. *Ans.* 3*ro.* 35*po.*

33. How much is the 10th part of 12 weeks' wages, at £2. 15*s.* 2½*d.* a week? *Ans.* £3. 6*s.* 3*d.*

34. If I get 2 cwt. 3 qrs. 4 lbs. sugar for a dozen pairs of shoes; what is that for each pair? *Ans.* 26*lbs.*

	£.	s.	d.				Answers.			
35.	223	17	6	÷	15	=	£14	18s.	6d.	
36.	92	8	11½	÷	25	=	3	13	11½	
37.	278	18	3¾	÷	27	=	10	6	7¼	
38.	7	11	2½	÷	32	=	0	4	8½	rem. 6½d.
39.	996	10	2	÷	56	=	17	15	10¾	
40.	174	8	0	÷	66	=	2	12	10	rem. 1s.
41.	246	3	11¼	÷	81	=	3	0	9¼	rem. 18¼d.
42.	129	7	9	÷	132	=	0	19	7¼	
43.	808	10	0	÷	210	=	3	17	0	
44.	641	14	10¾	÷	288	=	2	4	6¾	rem. 10¾d.
45.	541	7	9¾	÷	315	=	1	14	4¼	rem. 6s. 3d.

46.	111	15	7½	÷	73	=	£1	10s.	7½d.	
47.	554	9	3¼	÷	95	=	5	16	8¾	
48.	965	1	2¼	÷	157	=	6	2	11¼	
49.	204	16	10½	÷	239	=	0	17	1½	rem. 4s.
50.	884	0	3¾	÷	365	=	2	8	5¼	rem. 7¼d.
51.	79	6	5	÷	307	=	0	5	2	rem. 3d.
52.	997	0	0	÷	389	=	2	11	3	rem. 3s. 9d.
53.	317	10	4¾	÷	457	=	0	13	10¾	

54. If £12. 4*s*. 1½*d*. is paid for 63 yards of carpet; what is that per yard? *Ans*. 3*s*. 10½*d*.

55. When sugar sells at £3. 7*s*. 8*d*. the cwt.; what is that per lb.? *Ans*. 7½*d*.

56. Six pigs, weighing together 79 stones, are bought for £20. 4*s*. 10½*d*. Required the price per stone.

Ans. 5*s*. 1½*d*.

57. By what must we divide 1 mi. 1649 yds. 7 in. to get 1379 as the quotient? *Ans*. 2*yds*. 1*ft*. 5*in*.

58. If 609 quarters of wheat weigh 140 tons 5 cwt. 84 lbs.; what is the weight of a bushel?

Ans. 64*lbs*. 8*oz*.

XI. PROMISCUOUS EXERCISES.

1. A ham, weighing 15 lbs., cost 11*s*. 10½*d*. Required the cost per lb. *Ans*. 9½*d*.

2. Convert 527 dollars, each 4*s*. 6*d*., into £.

Ans. £118. 11*s*. 6*d*.

3. How many pairs of shoes, at 5*s*. 6*d*. a pair, may be got for £16. 4*s*. 6*d*.? *Ans*. 59.

4. Subtract £56. 14*s*. 3*d*. from 1000 crowns.

Ans. £193. 5*s*. 9*d*.

5. A ship has a cargo of bale goods measuring 4715 cubic feet. How many tons is that equal to, at 40 feet per ton? *Ans*. 117*t*. 17*cwt*. 2*qrs*.

6. Subtract 46 stones, each 14 lbs., from 1 ton.

Ans. 14*cwt*. 1*qr*.

7. A man having found a purse of 29 sovereigns, receives, on returning it to the owner, 4*s*. 6*d*. for every £. What is the amount of the reward?

Ans. £6. 10*s*. 6*d*.

8. Add together 75 guineas, 75 crowns, and 75 pence.

Ans. £97. 16*s*. 3*d*.

9. Add 264 lbs. 264 cwt. 264 tons, and 264 oz.

Ans. 277*t*. 6*cwt*. 2*qrs*. 8*oz*.

10. Reduce 13 cwt. 10 lbs. and 16 cwt. 10 oz. to whole numbers of one denomination. *Ans.* 23456 and 28682*oz.*

11. If a barrel of tar weigh 3 cwt.; what is the tons burden of a ship which carries 730 barrels?

Ans. 109*t.* 10*cwt.*

12. Reduce 17*s.* 8*d.* and £23. 16*s.* 6*d.* to whole numbers of one denomination. *Ans.* 106 and 2859*twopences.*

13. A person possessed of £200. spends the following portions of that sum; £13. 16*s.* 2½*d.*, £15. 13*s.* 6*d.*, £28. 4*s.*, and £10. 12*s.* 4¾*d.* Had he spent £9. 19*s.* less, how much should have remained?

Ans. £141. 12*s.* 10¾*d.*

14. Required the amount of 17 fares by a parliamentary train from London to Liverpool,—211 miles; the fare being a penny per mile. *Ans.* £14. 18*s.* 11*d.*

15. An oak tree contains 15 cubic feet of timber:—What is the cost of 29 such trees, at 3*s.* 4½*d.* the cub. ft.?

Ans. £73. 8*s.* 1½*d.*

16. Reduce 15 ft. 5 in. and 15 yds. 5 in. to whole numbers of one denomination. *Ans.* 185 and 545*in.*

17. If the brewing of 9 gallons of beer cost me 3*s.*; what does each pint cost? *Ans.* ½*d.*

18. Reduce 15 guineas and 17 dollars (each 4*s.* 6*d.*) to whole numbers of one denomination.

Ans. 630 and 153 *sixpences.*

19. How much paper, at 10*d.* per quire, is equal in value to 35 quires at 8*d.* per quire? *Ans.* 28 *quires.*

20. A land mile is 1760 yards, and a knot, or sea mile, is 2000 yards. How many knots correspond to 1400 land miles? *Ans.* 1232.

21. My watch, at noon, indicated 11 ho. 36 min. 32 sec. on the 13th of September; and again, at noon, on the 27th of the month, in the same place, it indicated 11 ho. 38 min. 10 sec. Required its daily gain.

Ans. 7*sec.*

22. How many lbs. of cherries, at 8*d.* a lb., are worth 9 bushels of apples at 3*s.* 6*d.* a bushel? *Ans.* 47*lbs.* 4 *oz.*

23. A man, whose weekly earnings are 17*s.* 6*d.*, saves a fifth part of that sum every fortnight. In what time will he have saved 50 guineas? *Ans.* 600wks.

24. In payment of a debt of £16. 5*s*. 6*d*., I gave ten mahogany chairs, valued at 19*s*. 6*d*. each, two arm-chairs, each 27*s*., and a chest of drawers. How much is reckoned for the drawers? *Ans*. £3. 16*s*. 6*d*.

25. A has 9 thousand 9 hundred and 9 pounds, B has 12 thousand 12 hundred and 12 pounds, and C has 15 thousand 15 hundred and 15 pounds. How much have they together? *Ans*. 39636*pounds*.

26. What cash must be given with 24 yards of cloth, at 8*s*. 3¾*d*., to pay for 6 cwt. sugar, at 38*s*. a cwt.?

Ans. £1. 8*s*. 6*d*.

27. An equivalent quantity of silk, worth 4*s*. 9*d*. per yard, is given in exchange for 5 yards of calico at 4½*d*., and 11 yards of Irish linen at 3*s*. 6*d*. Required the quantity of silk. *Ans*. 8*yds*. 2*qrs*.

28. I gave £106. 4*s*. for 32 pieces of cloth, each piece containing the same quantity, at 2*s*. 5½*d*. per yard. How many yards were in each piece? *Ans*. 27*yds*.

29. A offers to B 1350 feet of wood, at 2*s*. 11*d*. a foot, for 3 pipes of wine, each 126 gall. What price per gallon is offered? *Ans*. 10*s*. 5*d*.

30. I gave 26 dozen Port wine, and 23 dozen Sherry, the former worth 32*s*. 6*d*., and the latter worth 43*s*. a dozen, for an equivalent quantity of rum, valued at 14*s*. a gallon. How much rum did I receive? *Ans*. 131*g*.

31. A glass-cutter is paid 14½*d*. for every glass that he cuts successfully, and 7½*d*. for every one that he finishes in a certain degree imperfectly; moreover, he is fined 4*d*. for every one that he breaks. Of 6½ dozen which pass through his hands, 8 are of inferior execution, 5 are broken, and the rest are well finished. Required the amount of his earnings. *Ans*. £4. 1*s*. 10½*d*.

32. I can gain 4*s*. 6*d*. a day by a certain apparatus, as soon as it is got ready. A mechanic, whom I employ to construct it, offers to finish it in 7 days, for 5*s*. a day, —or, to prolong his daily labour, so as to have it done in 5 days, if I pay him 7*s*. 6*d*. a day. Which proposal should I accept, and what is the advantage?

Ans. Latter, 6*s*. 6*d*.

XII. SIMPLE PROPORTION.

There are two ways of comparing one number with another, to determine their relative magnitude. We may determine that the one exceeds the other by a certain difference; or we may determine that the one contains the other a certain number of times. Thus, in comparing 45 with 15, we may pronounce the former to be 30 more than the latter, or we may pronounce the former to be 3 times as great as the latter. In either case we express a relation between the two numbers. But the second mode of consideration determines that relation which we call *Ratio*. It used to be called, in the old books, the Geometrical Ratio; the difference being called the Arithmetical Ratio.

Definition.—RATIO means the relation which one quantity bears to another, as regards the number of times the one contains the other.

Now, if we equally increase or diminish the numbers 45 and 15, we shall not alter their relation as to difference; but we shall alter their relation as to ratio: 50 and 20 differ still by 30, but 50 is not 3 times 20.

Again; if we equally multiply or divide the numbers 45 and 15, we shall alter their relation as to absolute difference; but we shall not alter their relation as to ratio; 90 and 30 do not differ still by 30, but 90 is 3 times 30.

Ratio, then, is a relation independent of the absolute difference of numbers.

When the ratio of two numbers is equal to the ratio of two other numbers, the four numbers constitute what is called a *Proportion* or *Analogy*. Thus, the ratio of 45 to 15 is equal to the ratio of 36 to 12; for, in either case, the former number is 3 times the latter. Now, as the conventional sign expressive of ratio is two dots, we may write 45 : 15 = 36 : 12, and by these four terms we have constituted a Proportion. The

statement signifies that 45 contains 15 as often as 36 contains 12; but it is commonly read thus, 45 *is to* 15 *as* 36 *is to* 12.

It has long been customary to employ four dots instead of the sign of equality; thus, 45 : 15 :: 36 : 12.

Now, suppose a question to be put in this form; 45 is to 15 as 36 is to — what number? We may easily supply the answer, from the consideration, that as 45 is 3 times 15, so 36 is 3 times 12. The object of the Rule of Simple Proportion is to propose such questions; that is, it furnishes three terms of a proportion, and requires to determine the fourth.

Definition.—The Rule of SIMPLE PROPORTION proposes to find that quantity to which a given quantity bears the same Ratio as the former of two other given quantities bears to the latter.

We have indicated one method by which the solution of problems in Simple Proportion may be effected; that is, to divide the 3rd term by the quotient which arises from dividing the 1st by the 2nd. But the method usually practised is, to multiply the 2nd and 3rd terms together, and then to divide their product by the 1st. This is more convenient than the former; and it admits of easy demonstration. For, if instead of particular numbers we employ the letters a, b, and c, then, as every proportion signifies that $a \times c : a :: b \times c : b$, therefore, the product of the two inner or mean terms must always be equal to the product of the two outer or extreme terms; that is, $a \times b \times c = a \times c \times b$; and from this it is obviously deducible that the product of the means, divided by one extreme, gives the other extreme; so that the 4th term of any proportion is equal to the product of the 2nd and 3rd terms divided by the 1st. By special exemplification, we have 45 : 15 :: 36 : 12, the same as $15 \times 3 : 15 :: 12 \times 3 : 12$; and $15 \times 12 \times 3 = 15 \times 3 \times 12$, or $15 \times 36 = 45 \times 12$, or $15 \times 36 \div 45 = 12$.

The customary Rule for the solution of Problems in Simple Proportion is the following:—

Write, as the 3rd term of the analogy, that given quantity which is of the same nature as the required 4th term. If the 4th term is to be greater than the 3rd, let the greater of the two other given quantities be made the 2nd term; if less, let the less. Then multiply the 2nd and 3rd terms together, and divide their product by the 1st.

The first part of this Rule might be made less artificial in appearance, by saying, *Let the given quantities be so arranged that one of the two which are of like kind shall be to the other as the remaining quantity is to the required quantity.*

Examp. If 18 gall. of rum cost £13, what cost 54 gall.?

Here it is evident that the given price of 18 gall. must be to the required price of 54 gall. as 18 to 54; because price varies directly as the quantity purchased; therefore,—

18 gall. ⁚ 54 gall. ⁚⁚ £13 : the answer;
and £13 × 54 ÷ 18 = £39; consequently,
18 gall. ⁚ 54 gall. ⁚⁚ £13 ⁚ £39. *Ans.*

Examp. If 7 men perform a work in 48 hours, how many men can do it in 16 hours?

Here, the given number of men for 48 hours must be to the required number of men for 16 hours as 16 to 48, because the less the time is, the more the number of workmen must be: the number of agents employed varies inversely as the time they take.

16 hours ⁚ 48 hours ⁚⁚ 7 men ⁚ the answer;
7 men × 48 ÷ 16 = 21 men. *Ans.*

Problems in Simple Proportion may be resolved independently of an acquaintance with the usual formality of statement; and the pupil should frequently employ the demonstrative procedure of first principles in Exercises under this Rule.

Let the *data*, or conditions of the question, be written in a line, beginning with that quantity which is to be increased or diminished to form the answer; then, let that quantity be converted into the amount which will correspond with a unit of the other

given quantity; and lastly, let that amount be converted into another amount corresponding with the quantity about which the question is asked. This last amount will give the answer.

The following solutions of the first seven questions in the Manual will exemplify the method of first principles.

Examp. 1. Let the data, or things given, be written down, viz. £42 will buy 14 cwt.; then, as the price of 20 cwt. is required, and that directly depends on the price of 1 cwt., let the data be converted into the price for 1 cwt., viz., £42 ÷ 14 will buy 1 cwt.; lastly, let the price for 1 cwt, be converted into the price for 20 cwt., viz. £42. × 20 ÷ 14 will buy 20 cwt.; and 42 × 20 ÷ 14 = £60. *Ans.*

(2.) 96 lbs. measure 12 gal.

$$\frac{96 \text{ lbs.}}{12} \ldots 1 \text{ gal.}$$

$$\therefore \frac{96 \text{ lbs.} \times 23}{12} \ldots 23 \text{ gal.}$$

(3.) 40*s.* will buy 16 yds.

$$\frac{40s.}{16} \ldots 1 \text{ yd.}$$

$$\frac{40s. \times 10}{16} \ldots 10 \text{ yds.}$$

(4.) 35*s.* will buy 10 prs.

$$35s. \div 10 \ldots 1 \text{ pr.}$$

$$35s. \times 12 \ldots 12 \text{ prs.}$$
$$\overline{10}$$

(5.) 6 bu. weigh 372 lbs.

$$\frac{6 \text{ bu.}}{372} \ldots 1 \text{ lb.}$$

$$\frac{6 \text{ bu.} \times 496}{372} \ldots 496 \text{ lbs.}$$

(6.) 24 lbs. cost 18*s.*

$$\frac{24 \text{ lbs.}}{18} \ldots 1s.$$

$$\frac{24 \text{ lbs.} \times 12}{18} \ldots 12s.$$

(7.) 40 hrs. taken by 6 m.

$$40 \text{ hrs.} \times 6 \ldots 1 \text{ m.}$$

$$\frac{40 \text{ hrs.} \times 6}{15} \ldots 15 \text{ m.}$$

After the pupil has wrought the first nine Exercises, it may be considered desirable to acquaint him with the expedient called cancelling, which is frequently available to simplify the process of multiplying by one number and dividing the product by another. For example, 27 × 24 ÷ 18 is equivalent to 27 × 4 ÷ 3, the dividend and divisor having their common factor, 6, cancelled in each; and it is also equivalent to 3 × 24 ÷ 2, the dividend and divisor having 9 cancelled in each. The expedient consists in reducing the divisor and either factor of the dividend by dividing each of them by some measure common to both. That the result is not affected by cancelling may be easily demonstrated.

For, in the first place, any number may be converted into an infinite variety of equal magnitudes, with different denominations. Thus, the concrete number £35 = 7 Five £ Notes = 700 shill. = 140 crowns = 100 sums of 7 shillings, &c. So also the abstract number 72 = 36 pairs = 6 dozen = 8 nines = 144 halves, &c.

Accordingly, when two numbers are related as dividend and divisor, and can be reduced to one common denomination expressed by simpler figures, the division may be facilitated by such reduction. Thus 720 ÷ 48 = 60 dozen ÷ 4 dozen = 60 units ÷ 4 units = 15; so also 324 farthings ÷ 216 farthings = 81*d.* ÷ 54*d.* = 9 ninepences ÷ 6 ninepences = 3 sums (each 2*s.* 3*d.*) ÷ 2 sums (each 2*s.* 3*d.*) = 3 ÷ 2 = $1\frac{1}{2}$.

Let the pupil now work the first nine Exercises again, making use of cancelling. We subjoin solutions of the 8th and 9th, by first principles, and by statement, respectively.

(8.) 28 days are required by 27 men;
28 days × 27 . . . 1 man;

$$\frac{28 \text{ days} \times 27}{21} \quad . \quad . \quad . \quad 27 \text{ men};$$

$$\frac{28 \times 27}{21} = \frac{4 \times 27}{3} = 4 \times 9 = 36 \text{ da. } \textit{Ans.}$$

(9.) The ratio of 22*s.* to the answer must be the same as the ratio of 12 bot. to 15 bot.; therefore,

12 bot. : 15 bot. :: 22*s.* : *Ans.*

$$\frac{22 \times 15}{12} = \frac{22 \times 5}{4} = \frac{11 \times 5}{2} = \frac{55}{2} = 27s.\ 6d. \textit{ Ans.}$$

The exercises from Numbers 10 to 21, inclusive, have a compound quantity as the third term. The 18th may be here worked as an example.

32 ac. &c. ÷ 1560 is the quantity worth £1.; therefore, the quantity worth £1430 is $\frac{32 \text{ ac. \&c.} \times 1430}{1560}$

Cancelling first 10, and then 13, gives

$$\frac{32 \text{ ac. \&c.} \times 11}{12} = 29 \text{ ac. } 3 \text{ ro. } 3 \text{ po. } \textit{Ans.}$$

When the first twenty-one Exercises have been worked, some illustration will be necessary to enable the pupil to deal with compound quantities occurring

as first and second terms. In such instances, these terms cannot, in general, be conveniently employed in multiplication and division till they are reduced to whole numbers of one denomination.

Let us select for solution the 22nd question.

7 qrs. 3 bu. 2 pks. : 5 qrs. 6 bu. 3 pks. :: £154 : *Ans.*
or, 238 pks. : 187 pks. :: £154.

$$\frac{187 \times 154}{238} = \frac{187 \times 11}{17} = 11 \times 11 = £121. \textit{ Ans.}$$

We shall conclude this article with a Table of such properties of arithmetical analogy as are most frequently found to be useful.

If a, c, m, and n, represent any four abstract numbers, and are such that $a : c :: m : n$, then it may be demonstrated that

$a : m :: c : n$	$a + c : c :: m + n : n$
$c : a :: n : m$	$a - c : c :: m - n : n$
$a + c : a - c :: m + n : m - n.$	

Example.—For $a : c :: m : n$, put $36 : 27 :: 16 : 12$;

$36 : 16 :: 27 : 12$	$63 : 27 :: 28 : 12$
$27 : 36 :: 12 : 16$	$9 : 27 :: 4 : 12$
$63 : 9 :: 28 : 4.$	

EXERCISES.

1. If 14 cwt. of sugar cost £42.; what will 20 cwt. cost? *Ans.* £60.

2. If 12 gallons of oil weigh 96 lbs.; what is the weight of 23 gallons? *Ans.* 184*lbs.*

3. If 16 yds. of cloth cost 40*s.*; what cost 10 yds.? *Ans.* 25*s.*

4. If 10 pairs of gloves cost 35*s.*; what will a dozen pairs cost? *Ans.* 42*s.*

5. If 6 bushels of wheat weigh 372 lbs.; how many bushels weigh 496 lbs.? *Ans.* 8*bu.*

6. If 18*s.* is given for 24 lbs. of butter; what quantity is got for 12*s.*? *Ans.* 16*lbs.*

7. If 6 men perform a work in 40 hours; what time should 15 men take? *Ans.* 16*hrs.*

8. How many days will 21 men take to accomplish a work for which 27 men require 28 days? *Ans.* 36*da.*

9. If wine be sold at 22*s.* a dozen; what is the price of 15 bottles? *Ans.* 27*s.* 6*d.*

10. I have bought 50 eggs for 4*s.* 8½*d.*; what have I paid per dozen? *Ans.* 13½*d.*

11. Required the cost of 45 bushels of onions, at the rate of £6. 17*s.* 6*d.* for 30 bushels. *Ans.* £10. 6*s.* 3*d.*

12. If 28 yds. cost 10 guin.; what cost 35 yds.? *Ans.* £13. 2*s.* 6*d.*

13. How many yards of silk can be bought for 80*s.*, when the cost of 13 yds. 2 qrs. 3 na. is 48*s.*? *Ans.* 22*yds.* 3*qrs.* 1*na.*

14. A payment is made at the rate of six guineas and a half a year. How much is that for 5 months? *Ans.* £2. 16*s.* 10½*d.*

15. If 27 quires of paper cost 10*s.* 6*d.*; what is the cost of 45 quires? *Ans.* 17*s.* 6*d.*

16. A draper sells 10 dozen umbrellas, all at one price, for 20 guineas; how much does he receive for 70 of them? *Ans.* £12. 5*s.*

17. If 125 gall. Madeira wine pay duty £34. 7*s.* 6*d.*; how much is that per 100 gall.? *Ans.* £27. 10*s.*

18. What quantity of land is worth £1430.; if the value of 32 ac. 1 ro. 36 po. be £1560.? *Ans.* 29*ac.* 3*ro.* 3*po.*

19. Required the quantity of silver for £72., when the cost of 14 lbs. 6 oz. 15 dwt. is £45. *Ans.* 23*lbs.* 3*oz.* 12*dwt.*

20. If 3 cwt. 2 qrs. 8 lbs. of coffee are bought for £25.; how much of it costs £6.? *Ans.* 3*qrs.* 12*lbs.*

21. When sound travels 1 mi. 5 fur. 140 yds. in 8 seconds, what is its velocity per minute? *Ans.* 12*mi.* 6*fur.* 60*yds.*

22. Required the cost of 5 qrs. 6 bu. 3 pks. of beans; allowing £154. for 7 qrs. 3 bu. 2 pks. *Ans.* £121.

23. If 39 bushels of pears cost £9. 15*s.*; what quantity may be got for 10 guineas? *Ans.* 42*bu.*

24. If a house rated at £15. rent pay 18*s.* 1½*d.*, at what rent is a house rated which pays £1. 4*s.* 2*d.*?

Ans. £20.

25. How many bushels of potatoes are purchased for £4. 14*s*. 10½*d*., at the rate of £3. 18*s*. 4½*d*. for 57 bushels? *Ans*. 69*bu*.

26. A vessel, sailing at a uniform rate, has run 455 knots in 2 days 22 hours; how far has she run in 17 hours? *Ans*. 110½*knots*.

27. If 225 men perform a work in 17 da. 20 min.; how many men should do it in 15 da. 3 ho., reckoning 9 hours' work per day? *Ans*. 250*m*.

28. If an allowance of 2 lbs. 10 oz. to each man per day, will make a certain quantity of provisions serve a ship's crew for 35 days; how long would the same provisions last, at an allowance of 1 lb. 4 oz.? *Ans*. 73½*da*.

29. If 12 days suffice for the completion of a work, when the men are occupied 6 ho. 45 min. per day; in what time should the work be completed if they are occupied 7 hours in the day? *Ans*. 11*da*. 4*ho*.

30. I bought 112 gal. 2 qts. 1 pt. for £102.; for what must I sell 99 gal. 1 qt. 1 pt. of it, so as not to be a loser? *Ans*. £90.

31. Find the cost of 2 qrs. 24 lbs., at the rate of 20 guineas a ton. *Ans*. 15*s*.

32. If a piece of plate that weighs 7 oz. 9 dwt. 15 grs. cost £11. 12*s*. 2*d*.; how much of it costs £1. 13*s*. 2*d*.? *Ans*. 1*oz*. 1*dwt*. 9*grs*.

33. If I get 3*s*. 4½*d*. of profit on every guinea of outlay; what amount of outlay will gain £2. 16*s*. 3*d*.? *Ans*. £17. 10*s*.

34. Find the cost of 6 tons of iron, at 9*s*. 4½*d*. for 1¼ cwt. *Ans*. £45.

35. If 4 cwt. 1 qr. 26 lbs. of tobacco cost £43. 18*s*. 6*d*.; what quantity may be purchased for £4. 19*s*. 9*d*.? *Ans*. 2*qrs*. 1*lb*.

36. If £100. gain £5., what sum, at that rate, will gain £8. 2*s*. 6*d*.? *Ans*. £162. 10*s*.

37. A bankrupt owes his creditors £2960., and can pay them 12*s*. 6*d*. per £. How much would a person receive to whom he owes £641. 18*s*. 4*d*.?

Ans. £401. 3*s*. 11½*d*.

38. How many yards of cloth, at 17*s.* 5*d.* per yard, should be given for 41 E. ells, 4 qrs., at 19*s.* 9½*d.* per ell? *Ans.* 47*yds.* 2*qrs.*

39. If £10. 12*s.* 11*d.* be the wages of 14 men and 3 boys; how many persons will £11. 10*s.* 5*d.* pay, admitting that a man earns 5 times as much as a boy?
Ans. 15*men*, 4*boys.*

40. If £840., put into the bank for 146 days, gain £11. 15*s.* of interest; how long must £219. lie to gain the same interest? *Ans.* 560*da.*

41. If £72. 10*s.* 7½*d.* can purchase 5 pieces of cloth, each 33 yards; how many pieces of the same length will 50 guineas and a crown purchase? *Ans.* 3*pie.* 21*yds.*

42. By selling goods at £3. 14*s.* 6*d.* a cwt. which cost me 50*s.* a cwt. I gained 2 guineas; what quantity did I buy? *Ans.* 1*cwt.* 2*qrs.* 24*lbs.*

43. Bought 63 dozen of Sherry for £78. 10*s.*; and, having retained a certain quantity for private use, I sold the remainder at 29*s.* 6*d.* a dozen, and thereby cleared £4. 16*s.* 9*d.* over the reserve. What quantity did I retain? *Ans.* 6½ *dozen.*

44. A sets out from York, and travels 23 miles a day; B sets out from the same place, 4 days later, and follows him at 32 miles a day. In how many days from B's outset shall A be overtaken? *Ans.* 10⅘*da.*

XIII. COMPOUND PROPORTION.

Ratio may be constituted by a comparison in respect of more than one particular. If we compare 24 lbs. and 32 lbs., we may determine that the ratio of the one to the other is that of 24 to 32, or 3 to 4; but, if the comparison were between 24 lbs. Avoirdupois and 32 lbs. Troy, then, as the Troy lb. contains 5760 grains, and the Avoird. lb. 7000 such grains, the ratio must be that of 7000 grs. × 24 to 5760 grs. × 32, namely, a *compound ratio.*

A compound ratio is reduced to a simple ratio by performing the multiplication indicated in each term. But previous cancelling will often simplify the reduction. Thus, the above compound ratio is equal to $700 \times 3 : 576 \times 4 = 175 : 192$. The ratio, then, of 24 lbs. Avoird. to 32 lbs. Troy, in respect of 1 lb. of each, is that of 7000 to 5760; and their ratio in respect of the whole quantities, is that compounded of the ratios of 7000 to 5760 and 24 to 32; and by reduction to a simple ratio, we find that

24 lbs. Avoird. : 32 lbs. Troy :: 175 : 192.

Definition.—The Rule of COMPOUND PROPORTION proposes to find a quantity, to which a given quantity bears a given ratio;—that ratio being the compound or multiplied effect of two or more given ratios.

Examp. If 12 men reap 63 acres in 35 days; how many acres will 15 men reap, at that rate, in 20 days?

Here, 1 man would take 35 da. $\times$ 12, or 420 da. for 63 acres; 1 man would take 20 da. $\times$ 15, or 300 da. for the acres sought; therefore, If 63 ac. are reaped by one man in 420 days, how many acres will one man reap in 300 days? The question is now reduced to the form of Simple Proportion, and may be resolved by first principles, thus:

Quantity reaped in 1 day $= \frac{63 \text{ ac.}}{420}$; ditto reaped in 300 days $= \frac{63 \times 300}{420} = 9 \times 5 = 45$ ac. *Ans.*

The following is the usual method by statement:

$$\left.\begin{array}{rcl} 12 & : & 15 \text{ m.} \\ 35 & : & 20 \text{ da.} \\ \hline 420 & : & 300 \end{array}\right\} :: \quad 63 \text{ ac.} \quad : \quad 45 \text{ ac.}$$

The reason of the combined analogies may be thus stated. The given quantity, 63 ac., must be to the *required quantity* (as regards the number of reapers), as

12 m. : 15 m., and (as regards the number of days), as 35 da. : 20 da. Compounding these ratios, we obtain the simple analogy, 420 : 300 :: 63 ac. : 45 ac.

It is, however, expedient, after stating the terms, to arrange them as follows, in order to apply cancelling more conveniently:

$$\frac{63 \times 15 \times 20}{12 \times 35} = \frac{9 \times 5 \times 5}{1 \times 5} = 9 \times 5 = 45 \text{ ac.}$$

This procedure will be easily remembered from its similarity to that employed in Simple Proportion. As in Simple Proportion we multiply the 3rd term by the 2nd, and divide the product by the 1st; so in Compound Proportion we multiply the 3rd term by the continual product of all the 2nd terms, and then divide by the continual product of all the 1st terms.

Examp. How many days will 10 men require to manufacture 47½ reams of paper, working 12 hours a day; when 15 men take 9 days, working 11 hours a day, to make 52½ reams?

$$\left.\begin{array}{lll} 10 & : & 15 \text{ m.} \\ 12 & : & 11 \text{ ho.} \\ 52\frac{1}{2} & : & 47\frac{1}{2} \text{ re.} \end{array}\right\} :: 9 \text{ da.} : 11\tfrac{1}{2} \text{ da. } \textit{Ans.}$$

$$\frac{9 \times 15 \times 11 \times 190}{10 \times 12 \times 209} = \frac{9 \times 15 \times 11 \times 10}{10 \times 12 \times 11} = \frac{9 \times 15}{12} = \frac{45}{4}$$

The statement in the above solution, is thus determined. Days are required, therefore 9 days is the third term. Then we choose the numbers of men for comparison, and say, if 15 men take 9 days, 10 men will take more days; so far, then, as the numbers of workmen are concerned, the answer will be greater than the third term in the ratio of 10 to 15. Next, we choose the hours per day for comparison, and say, if 11 hours a day require 9 days, 12 hours a day will require fewer days; so far, therefore, as the duration of daily work is concerned, the answer will be less than the third term, *in the* ratio of 12 to 11. Lastly, we choose

for comparison the quantities of paper, and say, if $52\frac{1}{2}$ reams require 9 days, $47\frac{1}{2}$ reams will require fewer days; accordingly, so far as the quantities of paper are concerned, the answer will be less than the third term, in the ratio of $52\frac{1}{2}$ to $47\frac{1}{2}$, or, more conveniently, 209 quarters to 190 quarters.

The solution of the above question by first principles is as follows:

Given, $9\times15\times11$ hours for 1 man to make $52\frac{1}{2}$ reams; then, the 10th part of that time will suffice for 10 men to do the same; and reducing their hours to days, we have

$\frac{9\times15\times11}{10\times12}$ days for 10 men to make $52\frac{1}{2}$ reams;

$\frac{9\times15\times11}{10\times12\times209}$ days for 10 men to make 1 quarter ream;

$\frac{9\times15\times11\times190}{10\times12\times209}$ days for 10 men to make 190 ditto.

The teacher is recommended here, as in Simple Proportion, to require occasionally from the pupil solutions by first principles, as well as by the usual form of statement. Intellectual discipline will be thereby promoted, and the additional time thus expended will be well compensated by the acquirement of additional aptitude for calculation. A few more examples are subjoined, in which the two methods are employed alternately.

Examp. If 189 quarts of beer, worth 4*s.* 3*d.* a cask, be the allowance to 18 men for $8\frac{1}{2}$ days; how many quarts of beer, worth 4*s.* 1*d.* a cask, is a corresponding allowance to 28 men for $10\frac{1}{2}$ days?

$$\left.\begin{array}{l}49 : 51 \text{ pence.}\\18 : 28 \text{ men.}\\17 : 21 \text{ half days.}\end{array}\right\} :: 189 \text{ qts.} : 378 \text{ qts. } \textit{Ans.}$$

Examp. If, when wine is 3*s.* 6*d.* a bottle, 5 persons, drinking uniformly, are charged 21*s.* $10\frac{1}{2}$*d.* for $2\frac{1}{2}$ hours; *what is the price* per bottle, when 6 persons, drinking

with the same moderation, should be charged 27*s.* for 2½ hours?

Pence.		Pers.		Farth.	Qr. hrs.
42 per bottle, if	. .	5 be charged		1050	for 10.
42×5	. . . . .	1	. . .	1050	. 10.
$42\times5\times10$	. . . .	1	. . .	1050	. 1.
$\frac{42\times5\times10}{1050}$	. . . .	1	. . .	1	. 1.
$\frac{42\times5\times10\times1296}{1050\times6\times9}$	.	6	. . .	1296	. 9.

= 48*d.*, or 4*s.* per bottle. *Ans.*

Examp. If in a three-volume edition of a work, each volume contains 275 pages, each page 32 lines, and each line, on an average, 9 words; what is the average number of words per line in a two-volume edition of the same work, each volume containing 528 pages, and each page 30 lines?

$$\left.\begin{array}{r} 2 : 3 \text{ vol.} \\ 528 : 275 \text{ pa.} \\ 30 : 32 \text{ lines.} \end{array}\right\} :: 9 \text{ wo.} : 7\tfrac{1}{2} \text{ wo. } \textit{Ans.}$$

Examp. If, when wine is 9 prices per bottle, 15 persons are charged a certain sum for wine in a certain time; how many persons drinking half as rapidly should be charged twice the sum for thrice the time, when the wine is 10 prices per bottle?

Pers.		Sums.	Times.		Prices.
30 drinking ½ as fast, pay		1	for 1,	wine being	9
30×9	. . . . .	1	. 1,	. . .	1
$\frac{30\times9\times2}{3\times10}$	. . . . .	2	. 3,	. . .	10

= 18 persons. *Ans.*

Examp. If an engine of 10-horse power, working 8 hours a day, raise from a mine a weight of 52 tons in 6 days; *in what* time will an engine of 12-horse power,

working 3 days a week for 9 hours a day, raise from the same mine a weight of 175½ tons?

$$\left.\begin{array}{l} 12 : \text{10 h. p.} \\ 9 : \text{8 hours.} \\ 104 : \text{351 hf. tons.} \end{array}\right\} :: 6 \text{ da.} : 15 \text{ da. or 5 wks. } \textit{Ans.}$$

Examp. If I can sell 97 yards at 9*s.* 4*d.* for every 15 yards, when the article costs me 17*s.* 6*d.* per piece of 40 yards; what does the article cost me per piece of 48 yards, when, at the same rate of profit, I can sell 87 yards at 13*s.* 5*d.* for every 21 yards?

Pence.		Yds.	Pence.	Yds.
210	buying price of	40,	if 112 be selling price of	15
210×15	. . .	40,	. 112	1
$\frac{210\times15}{40}$	. . .	1,	. 112	1
$\frac{210\times15}{40\times112}$	. . .	1,	. 1	1
$\frac{210\times15\times48\times161}{40\times112\times21}$		48,	. 161	21

$= 258\frac{3}{4}d. = 21s.\ 6\frac{3}{4}d.$ *Ans.*

EXERCISES.

1. If 21 men can do 6 measures of work in 35 days; how many men can do 8 such measures in 28 days?
Ans. 35*m.*

2. If 8 horses can plough 25 acres in 3 days; how many acres can 30 horses plough in 5 days?
Ans. 156*ac.* 1*ro.*

3. In what time will 35 horses eat 104 bu. 2 pks. of oats, if 10 horses eat 19 bushels in 7 days? *Ans.* 11*da.*

4. If £225. gain £30. in 219 days; what will £240. gain, at the same rate, in 365 days? *Ans.* £53. 6*s.* 8*d.*

5. In what time will £225. gain £30. when £240. requires 365 days to gain £53. 6*s.* 8*d.*? *Ans.* 219*da.*

6. What sum will 15 men, working 9 hours a day, earn in 16 days, if 8 men, working 7 hours a day, earn £22. 15*s.* in 20 days? *Ans.* £43. 17*s.* 6*d.*

7. If the 6*d.* loaf weigh 35 oz. when wheat is 37*s.* a load; what should the 7*d.* loaf weigh when wheat is 29*s.* a load? *Ans.* $52\frac{1}{11}$*oz.*

8. If the carriage of 6 cwt. 2 qrs., for 124 miles, cost £3. 4*s.* 8*d.*; what weight should be carried 93 miles for £3. 0*s.* 7½*d.*? *Ans.* 8*cwt.* 14*lbs.*

9. A ship, with a crew of 32 men, has beef on board that will serve for 45 days, at a daily allowance of 2 lbs. to each man:—If it should then pick up the crew of another vessel, consisting of 16 men; what allowance would make the beef last 40 days? *Ans.* 1*lb.* 8*oz.*

10. If 27 packages of tea, each containing 8 lbs., cost £51.; what cost 52 packages, each 10 lbs., supposing 9 lbs. of the former to be worth 8 lbs. of the latter? *Ans.* £138. 2*s.* 6*d.*

11. If 6 men, of 3 times ordinary ability, earn 18 guineas in 21 days, working 9 hours a day; in how many days of 8 hours should 15 men, of 4 times ordinary ability, earn £24.? *Ans.* 9 *da.*

12. If beer, which is brewed with 5 pecks of malt to 18 gallons, cost 10*d.* a gallon, when malt is at 78*s.* a quarter; how much will the gallon of beer cost when brewed with 3 pecks of malt to 20 gallons, the price of malt being 65*s.* a quarter? *Ans.* 4½*d.*

13. Suppose 5 oxen to consume 156 Dutch lbs. of hay in 24 hours; how many avoirdupois lbs. would 14 oxen consume in 25 hours?—the lb. Dutch weighing 7600 grains troy, and the lb. avoirdupois 7000 ditto. *Ans.* 494*lbs.*

14. A., B., and C. lent me, each, a sum of money, for which I remunerated them at a uniform rate. A. lent me his money for 5 months, B. his for 9 months, and C. lent me £600. for 8 months. In refunding the loans, I paid for the use of A.'s money £6., for B.'s £7., for C.'s £16. Required the sums lent by A. and B. *Ans.* £360. by A.; £233. 6*s.* 8*d.* by B.

15. A. B. has goods worth 15*s.*, which he charges to me at 15*s.* 6*d.*, allowing 6 months' credit. I in return give him goods worth 40*s.*, charging him 2 guineas.

How long credit should I give him to make the transaction equitable? *Ans.* 9*mo.*

16. If 3 quires of paper cost as much as 40 quills, and 17 quills as much as 7 lead pencils, and 9 pencils as much as 4 sticks of sealing-wax; how many sticks of sealing-wax are worth 5 quires of paper?

Ans. $12\frac{3}{4}\frac{3}{5}$.

17. If 14 lbs. of sugar be worth 15 lbs. of raisins, 16 lbs. of raisins worth 7 lbs. of almonds, 5 lbs. of almonds worth 8 lbs. of currants, and 1 lb. of currants worth 10*d.*;—how much money should be given for 6 lbs. of sugar? *Ans.* 3*s.* 9*d.*

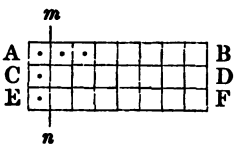
18. If A. goes 5 miles while B. goes 6, and B. goes 4 miles while C. goes 3, and C. goes 8 miles while D. goes 10;—how many miles will A. go while D. goes 18? *Ans.* 16 *mi.*

XIV. VULGAR FRACTIONS.

Nature of a Fraction.

If we take any whole number, and divide it by a greater number, the quotient will of course be less than 1, and is called a *Fraction.* Thus, if we take the number 3, and divide it by 8, the quotient is $\frac{3}{8}$, which signifies $3 \div 8$, or the 8th part of 3. Such a quotient merely continues to represent the division from which it results, unless we can assign to the upper number, or dividend, some value which is convertible to a greater number of an inferior name. Thus, if by $\frac{3}{8}$ we signify the 8th part of 3 lbs., each lb. = 16 oz., then, the 8th part of 3 lbs. is the 8th part of 3 times 16 oz., and the quotient may be expressed by 6 oz. as well as by $\frac{3\text{ lbs.}}{8}$. But if by $\frac{3}{8}$ we signify the 8th part of 3 farthings, then, as we have no money denomination of less value than farthings, the quotient continues in the fractional or *divisional form* $\frac{3}{8}f.$

Further: As $\frac{3}{8}$ represents the 8th part of 3 units, it must also represent 3 times the 8th part of 1 unit; as may be demonstrated by means of the annexed diagram.



Let there be three contiguous rows of squares, A B, C D, E F, each row representing 1 unit divided into eight equal parts. Then, the three dotted squares from A towards B, are 3 eighth parts of 1 unit; and the three dotted squares cut off by *m n*, are 1 eighth of the whole diagram, that is, 1 eighth of 3 units; therefore 3 eighths of 1, and 1 eighth of 3, are equivalent.

This interpretation of a Fraction may be vindicated also thus:—Since 1 unit = 8 eighths of 1 unit; therefore, 3 units = 24 eighths of 1 unit; and, therefore, the eighth parts of these equals must be equal, or, 1 eighth of 3 units = 3 eighths of 1 unit.

A Fraction, then, represents one of the equal parts into which its upper number is divided, or, one or more of the equal parts into which unity is divided. $\frac{2}{7}$ expresses 1 of seven equal parts into which 2 is divided, or 2 of seven equal parts into which 1 is divided.

Definition.—A FRACTION is a proportionate part of a whole, retaining the denomination of that whole.

When *unity* is the integer of which a fraction is understood to express a proportionate part, the lower term of the fraction is called the *Denominator*, as expressing the *denomination* of equal parts into which unity is supposed to be divided; and the upper term is called the *Numerator*, as expressing the *number* of equal parts taken.

Distinction of Fractions into Proper and Improper.

The *fractional form* may be assumed by a quantity whose value is unity, or higher than unity; because that form represents the relation of dividend and divisor,

and any number may have its value represented in that relation. Thus, 8 is the quotient of $8 \div 1$, or of $16 \div 2$, &c., therefore 8 may be represented by $\frac{8}{1}$ or $\frac{16}{2}$, &c. Thus also $23\frac{1}{2}$ may be expressed in the purely fractional form $\frac{47}{2}$. Moreover, 1 may be expressed by $\frac{2}{2}$, $\frac{3}{3}$, $\frac{6}{6}$, &c

A *Proper Fraction* is one whose numerator is less than its denominator, as $\frac{5}{9}$. An *Improper Fraction* is one whose numerator is not less than its denominator, as $\frac{4}{4}$, $\frac{12}{4}$, $\frac{10}{7}$. A *Mixed Number* consists of a whole number and a fraction, as $6\frac{4}{5}$.

An Improper Fraction is reduced to its proper form by performing the division which it indicates. Thus, $\frac{12}{4} = 3$; $\frac{10}{7} = 2\frac{3}{7}$.

A Mixed Number is reduced to an Improper Fraction by converting the given whole number into that kind of parts of unity which the given fraction expresses, and then combining the two fractional expressions in one sum. Thus, $2\frac{3}{7} = \frac{14}{7} + \frac{3}{7} = \frac{17}{7}$.

A Whole Number is reduced to an Improper Fraction, by merely writing 1 as a denominator. Thus $6 = \frac{6}{1}$. Or we may take any number we please for the denominator, if we multiply it by the given number for a numerator. Thus, $11 = \frac{44}{4}$.

EXERCISES.

1. *Conversion of* Improper Fractions *to* Whole *or* Mixed Numbers.

(1) $\frac{23}{4}$; (2) $\frac{19}{3}$; (3) $\frac{144}{18}$; (4) $\frac{89}{9}$; (5) $\frac{1000}{13}$.

Answers. $5\frac{3}{4}$; $3\frac{4}{5}$; 8; $26\frac{1}{3}$; $76\frac{12}{13}$.

2. *Conversion of* Whole *or* Mixed Numbers *to* Improper Fractions.

(1) $4\frac{3}{5}$; (2) $6\frac{4}{7}$; (3) $8\frac{1}{8}$; (4) 10; (5) $15\frac{10}{11}$.

(6) The value 11 may be represented by 3 *sixes;* so may the value 7 :—Required the expressions.

Answers. $\frac{14}{3}$; $\frac{34}{5}$; $\frac{65}{8}$; $\frac{10}{1}$; $\frac{175}{11}$; $\frac{66}{6}$; $6\frac{6}{6}$.

Reduction of a Simple Fraction to its Lowest Terms, by the Greatest Common Measure.

The value of any given fraction may be represented by any other numerator and denominator having the same ratio between them as those of the given fraction have. Thus $\frac{3}{4}$ is equivalent to $\frac{6}{8}$, or to $\frac{24}{32}$, &c., because $3 : 4$ as $3 \times 2 : 4 \times 2$, or as $3 \times 7 : 4 \times 7$. It is here presumed that the pupil will recognize the fractional form as expressing the ratio of the two terms, because they constitute a dividend and divisor. Accordingly, if the terms of a fraction be both multiplied or both divided by any number, the value of the fraction is not changed.*

Now, it is often both practicable and expedient to divide the terms of a fraction by some number that will measure them both, and thereby lessen each term. Thus, $\frac{14}{70}$ may be changed into an equivalent fraction composed of lower terms; for if we divide both terms by 2, we obtain $\frac{7}{35}$, and if we further divide them by 7, we obtain $\frac{1}{5}$. It may be remarked, that as 5ths are a denomination 14 times as great as 70ths, so in compensation, 42 parts are 14 times as many as 3 parts.

A practised arithmetician will generally be able to determine, by a little inspection, what divisor is applicable for the reduction of a fraction to lower terms; but we shall here exhibit a process, by which we may directly ascertain the Greatest Measure common to two numbers; as such a divisor will at once reduce the terms of a fraction to the lowest exact numbers.

Suppose it is required to determine the greatest common measure of 1173 and 1518. The principal steps of the investigation may be marked as follows :—

---

* Multiplying or dividing *both terms of a fraction* by the same number is, in effect, multiplying or dividing *the fraction* by *unity*. Thus $\frac{3}{4} \times \frac{7}{7} = \frac{21}{28}$; but $\frac{7}{7} = 1$; therefore $\frac{3}{4} = \frac{21}{28}$. This illustration may be given to the pupil after he has learned Division of Fractions.

Whatever is the greatest common measure of 1518 and 1173,

i. e. of 1173+345 and 1173,
is also *g. c. m.* of 345 and 1173,
i. e. of 345 and 345 + 345 + 345 + 138,
and also *g. c. m* of 345 and 138,
i. e. of 138+138+69 and 138,
and is also *g. c. m.* of 69 and 138,
that is, of 69 and 69 + 69,

but the greatest common measure applicable to this last form is evidently 69; therefore, 69 is the greatest common measure of 1173 and 1518; and on trial, it is found to be contained 22 times in the greater, and 17 times in the less.

The above process is virtually the same as the more convenient one exhibited in the margin; which consists in first dividing the greater of the given numbers by the less, then dividing the divisor by the remainder, and continuing to divide similarly until an exact divisor is found. That exact divisor is the greatest common measure of the given numbers.

```
1173)1518(1
 1173
 ----
345)1173(3
 1035
 ----
138) 345(2
  276
  ---
 69)138(2
  138
  ---
```

When it is required to find the greatest measure common to three quantities, we take the third number and the greatest common measure of the first and second, as two quantities whose greatest common measure is that required.

Thus, to find the g. c. m. of 1173, 1518, and 713:—

The g. c. m. of the first two numbers is found as above, to be 69; and the g. c. m. of 69 and 713 is found, as in the margin, to be 23. Therefore, 23 is the g. c. m. of the three given numbers.

```
69)713(10
 690
 ---
23) 69(3
 69
 --
```

The pupil having performed the Exercises in Section 3rd, will then be prepared to reduce a fraction to its

lowest terms, by employing the greatest common measure of the numerator and denominator.

Thus, to reduce to its lowest terms the fraction $\frac{8}{12}\frac{4}{8}$, we find the g. c. m. of the numerator and denominator to be 17, which converts the fraction into $\frac{8}{12}$.

EXERCISES.

3. *To find the* Greatest Common Measure *of two or more* Whole Numbers.

(1) 161 and 299; (2) 1736 and 341; (3) 728 and 4536; (4) 3320 and 1577; (5) 969 and 912; (6) 257 and 2049;

(7) What length of a line will apply as the greatest common measure to three distances, which are 12702, 24186, and 10237, feet?

Answers. 1. 23; 2. 31; 3. 56; 4. 83; 5. 57; 6. 1; 7. 29 ft.

4. *Reduction of a* Simple Fraction *to its* Lowest Terms.

Reduce to lowest terms (1) $\frac{12}{30}$; (2) $\frac{28}{42}$; (3) $\frac{64}{96}$; (4) $\frac{56}{91}$; (5) $\frac{68}{85}$; (6) $\frac{105}{126}$; (7) $\frac{840}{3465}$; (8) $\frac{855}{2357}$; (9) $\frac{1164}{1464}$; (10) $\frac{6828}{7128}$; (11) $\frac{9580}{8280}$; (12) $\frac{2689}{2419}$;

(13) Express $\frac{709}{1739}$ in the highest denomination for which it contains an exact numerator.

Answers. 1. $\frac{2}{5}$; 2. $\frac{4}{7}$; 3. $\frac{28}{151}$; 4. $\frac{8}{13}$; 5. $\frac{68}{82}$; 6. $\frac{5}{6}$; 7. $\frac{4}{9}$; 8. $\frac{5}{21}$; 9. $\frac{7}{9}$; 10. $\frac{19}{24}$; 11. $\frac{5}{17}$; 12. $\frac{1}{9}$; 13. $\frac{17}{37}$.

Reduction to an Assigned Denominator.

We may convert numerators of one denomination into numerators of another, by finding what multiplier or divisor will convert the given denominator into the required one, and using that multiplier or divisor for both terms of the given fraction.

Thus, to reduce 8-*twelfths* to an equivalent number

of *thirty-sixths*: The given denominator, 12, is converted into the required denominator, 36, by multiplying by 3; therefore, $\frac{8}{12} = \frac{8\times3}{12\times3} = \frac{24}{36}$ *Ans.*

When the given denominator is not an exact part or multiple of that required, the following method will be easier at this stage of the pupil's progress.

Suppose it is required to reduce $\frac{7}{9}$ to 12ths:—7 units = 84 twelfths of a unit; therefore, the 9th of 7 units = $9\frac{1}{3}$ twelfths.

This result shows the required fraction to be a *complex* one, viz. $\frac{9\frac{1}{3}}{12}$; and it must remain complex, if the denomination of 12ths is to be retained.

EXERCISES.

5. *Reduction to an* Assigned Denominator.

Express (1) $\frac{7}{12}$ in 60ths; (2) 76 in 4ths; (3) $\frac{9}{13}$ in 52ds; (4) $\frac{25}{28}$ in 420ths; (5) 9 in 3ds; (6) $\frac{4}{5}\frac{5}{9}$ in 12ths; (7) 200 in 18ths; (8) $4\frac{1}{2}$ in 6ths; (9) $\frac{11}{21}$ in 14ths; (10) $7\frac{2}{3}$ in 15ths; (11) $\frac{7}{9}\frac{8}{0}\frac{9}{4}$ in 20ths; 12. In $523\frac{1}{2}$ inches—how many 8th parts of an inch?

Answers. 1. $\frac{25}{60}$; 2. $\frac{304}{4}$; 3. $\frac{40}{52}$; 4. $\frac{375}{420}$; 5. $\frac{27}{3}$; 6. $\frac{10}{18}$; 7. $\frac{3400}{18}$; 8. $\frac{27}{6}$; 9. $\frac{8}{14}$; 10. $\frac{114}{15}$; 11. $\frac{15}{20}$; 12. 4188 eighths.

Reduction to the Least Common Denominator, by the Least Common Multiple.

For convenience in comparing fractions, we should have them expressed by one common denominator. Thus, it is not at once obvious whether $\frac{4}{9}$ or $\frac{5}{12}$ is the greater in value; but if each fraction is reduced to 36ths, then the expressions $\frac{16}{36}$ and $\frac{15}{36}$ indicate that $\frac{4}{9}$ is *of higher* value than $\frac{5}{12}$.

Any assemblage of fractions may be reduced to a common denominator; and it is often an easy matter to determine, by an inspection of the given denominators, for what common denominator they can be exchanged. If the given fractions are $\frac{1}{2}$, $\frac{3}{8}$, and $\frac{5}{9}$, it is easy to discern that each can be conveniently reduced to 72nds. If the pupil, however, feel at a loss to decide in this way, he may always be sure that the continued product of the given denominators will constitute a common denominator, to which all the numerators may be conformed by the process pertaining to Section 5th.

But the continued product of the denominators does not generally supply the *least* common denominator that will suit the given fractions. Accordingly, a method has been investigated for determining the Least Common Multiple of several denominators, that is, the least number which contains each denominator an exact number of times.

The continued product of 6, 8, and 9, is 432, which is a multiple, therefore, of each of the factors; but 216 is also a multiple of each; so is 144, and so is 72. Now 72 is their *least* common multiple; that is, no number less than 72 admits of being exactly measured by 6, 8, and 9, severally.

Let us inquire, then, how the least common multiple of several numbers may be determined; first, supposing it required to find the least common multiple of 18 and 24.

The greatest common measure of these numbers is 6, and, therefore, the smallest exact terms in which their ratio may be expressed are 3 and 4. Now, when we state 18 : 24 :: 3 : 4, we know that the product of the extremes equals that of the means; or that 3 is the smallest exact number of twenty-fours equalling an exact number of eighteens. Accordingly, 24×3, or 18×4, viz. 72, must be the least common multiple of 18 and 24.

Again, suppose it is required to find the least common multiple *of 18, 24,* and 27.

The l. c. m. of the first two numbers is 72, as already found. Now, as 72 is not also a multiple of 27, the l. c. m. of the three numbers must be that of 72 and 27. Accordingly, the greatest com. meas. of 72 and 27 is 9; and, therefore, the least terms of the ratio of 72 to 27 are 8 and 3; consequently 72×3, or 27×8, $= 216$, is the least com. mul. required.

Note.—When of the given numbers one happens to be an exact multiple of the other, that other may evidently be cancelled altogether. If, for example, the l. c. m. of 4, 6, 8, and 9, be required, it is manifest that whatever is a multiple of 8 must be also a multiple of 4; therefore, the l. c. m. of 6, 8, and 9, will be that of the four given numbers.

As a practical rule, then, for finding the least common multiple of two or more whole numbers:—Determine, by observation or trial, the greatest common measure of any two of the given numbers; divide one of them by it, and multiply the other by the quotient; the result is the l. c. m. of the two selected numbers. Then take that result and another of the given numbers, and repeat the process, &c.

Thus, to find the least common multiple of 25, 36, 40, and 32:—

$5)25, 40$; $\quad 8)200, 32$; $\quad 4)800, 36$;

$25 \times 8 \qquad 200 \times 4 \qquad 800 \times 9 = 7200$ *Ans.*

To apply this process to its usual purpose, let it be required to represent the following fractions by equivalent ones having a common denominator: $\frac{7}{10}$, $\frac{5}{18}$, $\frac{4}{15}$, $\frac{13}{24}$.

$2)10, 18$; $\quad 15)90, 15$; $\quad 6)90, 24$;

$10 \times 9 \qquad 90 \times 1 \qquad 90 \times 4 = 360.$

The l. c. m. of the denominators is 360; therefore, according to Section 5th,

$$\frac{7}{10} = \frac{252}{360} \qquad \frac{4}{15} = \frac{96}{360}$$

$$\frac{5}{18} = \frac{100}{360} \qquad \frac{13}{24} = \frac{195}{360}$$

EXERCISES.

6. *To find the* Least Common Multiple *of two or more* Whole Numbers.

Required the least common multiple of (1) 28 and 77; (2) 54 and 144; (3) 17 and 21; (4) 16, 18, and 21; (5) 14, 21, and 24; (6) 9, 8, and 7; (7) 36, 40, 32, and 18; (8) 42, 63, 15, and 20; (9) 26, 12, 39, and 52. (10) What is the smallest number that can be counted by an exact number of sevens, dozens, and scores?

Answers. 1. 308; 2. 432; 3. 357; 4. 1008; 5. 168; 6. 504; 7. 1440; 8. 1260; 9. 156; 10. 420.

7. *Reduction of Fractions to the* Least Common Denominator.

(1) $\frac{7}{8}$ and $\frac{3}{10}$ = $\frac{35}{40}$ and $\frac{12}{40}$; (2) $\frac{9}{14}$ and $\frac{1}{21}$ = $\frac{27}{42}$ and $\frac{2}{42}$; (3) $\frac{1}{12}$, $\frac{23}{24}$, and $\frac{7}{16}$ = $\frac{6}{72}$, $\frac{69}{72}$, and $\frac{21}{72}$; (4) $\frac{1}{18}$, $\frac{3}{20}$, and $\frac{11}{36}$ = $\frac{20}{360}$, $\frac{54}{360}$, $\frac{220}{360}$; (5) $\frac{5}{6}$, $\frac{7}{9}$, $\frac{8}{15}$, and $\frac{1}{25}$ = $\frac{375}{450}$, $\frac{350}{450}$, $\frac{240}{450}$, and $\frac{18}{450}$.

Addition and Subtraction of Vulgar Fractions.

As we cannot tell the sum or the difference of 23 crowns and 43 pence, before reducing the two quantities to some common denomination; so we cannot tell the sum or the difference of 3 *fourths* and 5 *sixths*, before reducing the fractions to a common denominator. And when such reduction is performed, it is manifest that as the denominators express merely a *denomination* of the quantities, the change made by the addition or subtraction of the fractions applies to their numerators only.

Thus, if it be required to find the sum of $\frac{7}{16} + \frac{5}{18} + \frac{4}{15} + \frac{13}{14}$; we obtain from the last example $252 + 100 + 96 + 195 = 643$, the sum of the proper numerators; and $\frac{643}{360} = 1\frac{283}{360}$. *Ans.*

In like manner, if it be required to find the difference

of $\frac{3}{5}$ and $\frac{4}{7}$, we take the difference of $\frac{3}{5}\frac{1}{0}$ and $\frac{4}{5}\frac{0}{0}$, which is $\frac{1}{5}\frac{9}{0}$. *Ans.*

When any of the aggregate parts proposed for addition are whole or mixed numbers, add the fractions by themselves, and afterwards combine their sum with that of the given integral quantities.

We subjoin two Subtraction examples, in solving which we employ an expedient requiring illustration.

Find the difference between 17 and $2\frac{3}{4}\frac{1}{0}$, and also between $56\frac{1}{8}$ and $28\frac{1}{7}$.

In the first example, as there are no 26ths expressed in the minuend, conceive $\frac{2}{2}\frac{6}{6}$ to be added to the minuend, and the equivalent 1 to be added to the subtrahend; then the upper number is $17\frac{2}{2}\frac{6}{6}$, and the under $3\frac{3}{4}\frac{1}{0}$; the subtraction is then easy.

17	$56\frac{1}{8}$
$2\frac{3}{4}\frac{1}{0}$	$28\frac{1}{7}$
$14\frac{2}{2}\frac{5}{6}$	$27\frac{1}{5}\frac{7}{6}$

In the second example, we cannot subtract $\frac{4}{5}\frac{9}{0}$ from $\frac{3}{5}\frac{1}{0}$; but if we increase the minuend by $\frac{2}{5}\frac{5}{0}$, and the subtrahend by 1, we can easily take $29\frac{4}{5}\frac{9}{0}$ from $56\frac{7}{5}\frac{7}{0}$.

EXERCISES.

8. *Addition of Vulgar Fractions.*

(1) $\frac{5}{8}+\frac{1}{4}$; (2) $\frac{8}{11}+\frac{7}{11}$; (3) $\frac{3}{5}+\frac{3}{10}+\frac{7}{15}$;
(4) $\frac{3}{8}+\frac{27}{50}+\frac{1}{7}$; (5) $\frac{1}{16}+\frac{1}{16}+\frac{1}{21}+\frac{1}{30}+\frac{1}{84}$;
(6) $\frac{1}{30}+\frac{17}{85}+\frac{24}{45}+\frac{101}{110}$; (7) $26\frac{1}{10}+\frac{8}{15}+6\frac{5}{12}$;
(8) $2\frac{1}{8}+3\frac{3}{8}+\frac{9}{14}+\frac{1}{15}+6\frac{17}{30}$;
(9) $62\frac{1}{18}+59\frac{5}{10}+5\frac{4}{7}+56\frac{1}{12}+4\frac{1}{3}$.

Answers. 1. $\frac{7}{8}$; 2. $\frac{9}{11}$; 3. $1\frac{7}{10}\frac{9}{0}$; 4. 1; 5. $\frac{1}{3}\frac{3}{4}\frac{7}{0}$; 6. $2\frac{3}{45}$; 7. $33\frac{1}{30}$; 8. $13\frac{17}{16}\frac{7}{8}$; 9. $189\frac{3}{14}\frac{1}{0}$.

9. *Subtraction of Vulgar Fractions.*

(1) $\frac{7}{12}-\frac{5}{12}$; (2) $\frac{17}{30}-\frac{1}{30}$; (3) $\frac{13}{14}-\frac{13}{18}$; (4) $\frac{13}{18}-\frac{11}{15}$;
(5) $8\frac{1}{2}-4\frac{1}{4}$; (6) $63\frac{11}{18}-23\frac{13}{36}$; (7) $76\frac{1}{4}-9$;
(8) $13\frac{1}{8}-\frac{3}{8}$; (9) $121-\frac{1}{18}$; (10) $1-\frac{5}{18}$; (11) $8\frac{1}{3}-7\frac{2}{3}$.

Answers. 1. $\frac{1}{6}$; 2. $\frac{4}{6}\frac{9}{0}$; 3. $\frac{1}{5}\frac{3}{6}$; 4. $\frac{1}{3}\frac{1}{5}$; 5. $4\frac{1}{4}$; 6. $39\frac{1}{1}\frac{7}{8}\frac{3}{6}$; 7. $67\frac{1}{4}$; 8. $13\frac{1}{8}$; 9. $120\frac{1}{3}\frac{3}{6}$; 10. $\frac{7}{18}$; 11. $\frac{1}{15}$.

Multiplication of Vulgar Fractions.

Suppose it is required to multiply $\frac{3}{8}$ by 4. Then, from what has been shown of Addition, it must be evident that $\frac{3}{8} \times 4$, or $\frac{3}{8}$ taken 4 times, is $\frac{12}{8}$. Here the operation has been made to affect only the numerator. But it is manifest, from the product $\frac{3 \times 4}{8}$ being equal to $\frac{3}{\frac{8}{4}}$, that dividing the denominator will be, in some instances, a preferable mode of multiplying a fraction by a whole number.

Suppose it is required to multiply $\frac{9}{30}$ by $\frac{1}{3}$. The multiplicand is to be taken $\frac{1}{3}$ of once, or it is to be divided into 3 equal parts, and one of these is to be taken. Accordingly, we shall have divided $\frac{9}{30}$ by 3, if we make its denominator 30ths, so that $\frac{9}{30} \times \frac{1}{3} = \frac{9}{\frac{30}{3}}$; or we shall have divided $\frac{9}{30}$ by 3 if we make its numerator the 3rd part of 9, so that $\frac{9}{30} \times \frac{1}{3} = \frac{\frac{9}{3}}{30}$. It appears, then, that dividing the numerator will be, in some instances, a preferable mode of multiplying a fraction by a primitive fraction.*

Suppose it is required to multiply $\frac{3}{8}$ by $\frac{1}{4}$. We are required to take 4 times the 9th of $\frac{3}{8}$; and as $\frac{1}{9}$ of $\frac{3}{8}$ is $\frac{3}{72}$, therefore 4 times the 9th must be $\frac{4}{72}$ or $\frac{1}{18}$.

It hence appears that the multiplication of fractions is effected when we have placed the product of the given numerators over that of the given denominators.

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* A *primitive fraction* is one whose numerator is 1, as $\frac{1}{3}$. It is the reciprocal of a whole number; thus, $\frac{1}{3}$ is the reciprocal of 3; $\frac{1}{5}$ is the reciprocal of 5.

The multiplication of any number by a primitive fraction is resolvable into pure division. "*Multiplication*, properly so called," says Newton, "is that which is made by Integers, as seeking a new quantity, so many times greater than the multiplicand as the multiplier is greater than unity. But, for want of a better word, *Multiplication* is also made use of in Fractions and Surds, to find a new quantity, in the same *ratio* (whatever it be) to the multiplicand as the multiplier has to unity."—Arithm. *Univers.*, translated by Raphson.

Thus, $\frac{6}{8} \times 4 = \frac{6}{8} \times \frac{4}{1} = \frac{24}{8}$; also $\frac{9}{10} \times \frac{1}{2} = \frac{9}{20}$; also $\frac{2}{3} \times \frac{1}{3} = \frac{2}{9}$.

The numerators and denominators of fractions, in multiplication, often admit the application of cancelling, on the principle of reduction to lower terms; thus,

$$\frac{2}{10} \times \frac{25}{27} = \frac{2 \times 25}{10 \times 27} = \frac{1 \times 5}{2 \times 9} = \frac{5}{18}.$$

The word *of* is often used between fractions, as an equivalent for the sign of multiplication, in which case the multiplier precedes the multiplicand. Thus, $\frac{3}{5}$ *of* $\frac{7}{10} = \frac{7}{10} \times \frac{3}{5}$. It seems needless to give such a form as $\frac{3}{5}$ of $\frac{7}{10}$, the name of Compound Fraction, as many writers on Arithmetic have done.

When Mixed Numbers are proposed for multiplication, the general mode of dealing with them is, first to convert them into Improper Fractions, and then to employ the operation already illustrated. But when the integral part of a given factor is large, it is generally expedient to find the entire product by means of two partial products; thus, for the 16th Exercise—

$$8375\tfrac{3}{5}\tfrac{6}{8}\tfrac{5}{2} \times 435 = 8375 \times 435 + \tfrac{3}{5}\tfrac{6}{8}\tfrac{5}{2} \times 435.$$

$$8375 \times 435 = 3643125$$

$$\tfrac{3}{5}\tfrac{6}{8}\tfrac{5}{2} \times 4\tfrac{3}{1}\tfrac{5}{0} = \frac{365 \times 5}{6} = 304\tfrac{1}{6}$$

Ans. $3643429\frac{1}{6}$

Note.—It will be useful for the pupil to recognise readily, that when a fraction is multiplied by a number equal to the denominator, the product is equal to the numerator; thus, 4 times $\frac{3}{4}$ is 3.

EXERCISES.

10. *Multiplication of Vulgar Fractions.*

(1) $\frac{3}{5} \times \frac{9}{10}$; (2) $\frac{6}{7} \times 1\frac{1}{2}$; (3) $\frac{4}{5} \times \frac{6}{7} \times \frac{1}{2}$;
(4) $\frac{8}{9}$ of $\frac{9}{10} \times \frac{10}{11}$; (5) $\frac{3}{10}$ of $1\frac{2}{7}$ of $1\frac{1}{6}$ of $\frac{8}{27}$;
(6) $\frac{7}{8} \times 5$; (7) $\frac{5}{6} \times 4$; (8) $\frac{5}{8} \times 3$;
(9) $\frac{1}{3} \times \frac{7}{9} \times \frac{8}{9} \times 3$; (10) $2\frac{1}{2} \times 4\frac{3}{4}$;

(11) $3\frac{3}{8} \times 9\frac{1}{4} \times \frac{1}{21}$;
(12) $6\frac{7}{8} \times 16 \times 7\frac{7}{10} \times 1\frac{3}{4}$;
(13) $469 \times 26\frac{3}{8}$;
(14) $574 \times 100\frac{10}{101}$;
(15) $365\frac{15}{38} \times 97$;
(16) $8375\frac{4}{5}\frac{5}{2}\frac{1}{3} \times 435$;
(17) $69\frac{3}{8} \times 65\frac{8}{20}$;
(18) $147\frac{5}{21} \times 44\frac{1}{16} \times 14$.

Answers. 1. $\frac{5}{15}$; 2. $\frac{4}{5}$; 3. $\frac{4}{21}$; 4. $\frac{8}{17}$; 5. $\frac{1}{17}$.
6. $4\frac{4}{5}$; 7. $\frac{3}{5}$; 8. $2\frac{1}{2}$;
9. $1\frac{11}{15}$; 10. $11\frac{7}{8}$; 11. $15\frac{3}{4}$;
12. $786\frac{1}{2}$; 13. $123697\frac{7}{8}$; 14. $57968\frac{15}{21}$;
15. $35456\frac{27}{28}$; 16. $3643429\frac{1}{8}$; 17. $4540\frac{10}{15}$;
18. $90827\frac{1}{2}$.

Division of Vulgar Fractions.

Required to divide $\frac{4}{7}$ by 3.

If by this it is meant to divide $\frac{4}{7}$ into 3 equal parts, then each part is just $\frac{1}{3}$ of $\frac{4}{7}$, or $\frac{4}{21}$.

If the meaning is, to find how often 3 is contained in $\frac{4}{7}$, then as 3 is contained in 6 exactly 2 times, therefore in the 7th part of 6 it is contained the 7th part of 2 times, or $\frac{2}{7}$ of a time.

In the former case the answer is $\frac{4}{21}$ of *a unit*, in the latter case $\frac{2}{7}$ of *once*.

But the best general form of illustrating fractional division is by conceiving the dividend and divisor to have a common denominator equal to the product of the given denominators.

Thus, $\frac{7}{9} \div \frac{1}{3} = \frac{21}{27} \div \frac{9}{27} = 21 \div 9 = 2\frac{1}{3}$.

$\frac{5}{6} \div \frac{7}{8} = \frac{40}{48} \div \frac{42}{48} = 40 \div 42 = \frac{40}{42}$.

In these examples, it is evident that there is no necessity to express the common denominator, just as there is no necessity to express the common denomination of *sixpences* or *pence*, to which we might reduce £20. 10*s*. and 12*s*. 6*d*. in order to find how often the latter sum is contained in the former.

Hence the rule for fractional division may be thus stated :—

Multiply the dividend by the reciprocal of the divisor, *i. e.* by the divisor inverted.

Accordingly, $\frac{5}{6} \div \frac{7}{8} = \frac{5}{6} \times \frac{8}{7} = \frac{5 \times 4}{3 \times 7} = \frac{20}{21}$.

When Mixed Numbers are given, and the integral portion is large, we may multiply the dividend and divisor by some common multiple of the denominators, which will cancel the fractions, and then divide the one result by the other.

Thus, to divide $943\frac{2}{15}$ by $47\frac{3}{10}$.

$47\frac{3}{10}$	$943\frac{2}{15}$
10	15
473	14147
3	2
1419)	28294

$19\frac{2}{1419} = 19\frac{2}{3}$. *Ans.*

In this example we have multiplied each given number by the least common multiple of 10 and 15, viz. 30, which, for convenience, we have applied to the divisor by the two factors 10 and 3, and to the dividend by the two factors 15 and 2.

Recognising the fractional form itself as representative of division, the pupil will now experience no difficulty in reducing what are called complex fractions to their simplest form.

For example, $\frac{2\frac{1}{5}}{7} = 2\frac{1}{5} \div 7 = 14 \div 35 = \frac{14}{35} = \frac{2}{5}$.

So also, $\frac{\frac{1}{2}}{1\frac{3}{5}} = \frac{1}{2} \div 1\frac{3}{5} = 2 \div 11 = \frac{2}{11}$.

Such fractions sometimes arise as remainders from

division, which is the case in the latter of the two subjoined methods of dividing $1375\frac{1}{5}$ by 12.

$$\begin{array}{rr} 12 & 1375\frac{1}{5} \\ 5 & 5 \\ \hline 60) & 6876 \\ & \hline & 114\frac{3}{5} \end{array} \qquad \begin{array}{r} 12)1375\frac{1}{5} \\ \hline 114\frac{3}{5} \end{array}$$

By the former method the remaining fraction is $\frac{3}{60} = \frac{3}{5}$. By the latter method it is

$$\frac{7\frac{1}{5}}{12} = 7\tfrac{1}{5} \div 12 = 36 \div 60 = \tfrac{3}{5}.$$

The latter method is often preferable, when the divisor is a whole number.

EXERCISES.

11. *Division of Vulgar Fractions.*

(1) $\frac{7}{12} \div \frac{9}{16}$; (2) $\frac{1}{20}\frac{3}{0} \div \frac{1}{40}\frac{7}{0}$; (3) $\frac{3}{15} \div \frac{7}{16}$;
(4) $\frac{2}{7} \div \frac{7}{8}$; (5) $\frac{2}{7} \div 3$; (6) $\frac{3}{15} \div 4$;
(7) $4 \div 1\frac{3}{5}$; (8) $1 \div \frac{1}{10}$; (9) $1\frac{1}{8} \div \frac{3}{8}$;
(10) $\frac{3}{8} \div 1\frac{1}{8}$; (11) $\frac{4}{6}\frac{2}{3}$ of $\frac{3}{15} \div \frac{1}{18}$;
(12) $4745 \div 56\frac{3}{16}$; (13) $35\frac{3}{7} \div 372$;
(14) $5304\frac{1}{15} \div 9$; (15) $171 \div 304$;
(16) $2387\frac{1}{12} \div 7$; (17) $3011\frac{1}{5} \div 8$;
(18) $5362\frac{1}{16} \div 38\frac{1}{14}$.

Answers. 1. $1\frac{1}{27}$; 2. $\frac{1}{17}$; 3. $1\frac{1}{3}\frac{3}{3}$; 4. $\frac{4}{45}$;
5. $\frac{5}{11}$; 6. $\frac{3}{15}$; 7. $7\frac{1}{2}$; 8. 10; 9. $1\frac{17}{24}$; 10. $\frac{6}{11}$;
11. $\frac{2}{9}\frac{8}{5}$; 12. $84\frac{1}{3}$; 13. $\frac{3}{21}$; 14. $589\frac{5}{16}\frac{5}{5}$; 15. $\frac{9}{16}$;
16. $341\frac{1}{84}$; 17. $376\frac{4}{5}$; 18. $137\frac{4}{5}\frac{2}{5}\frac{2}{4}$.

Reduction, &c. of Concrete Fractions.

Let it be required to reduce £$\frac{9}{17}\frac{5}{4}$ to pence.

As £. are reduced to pence by multiplying by 20

and 12 successively, or at once by 240, we therefore multiply the given fraction thus—

$\frac{5}{1\,4} \times \frac{2\,0}{1} \times \frac{1\,2}{1}$ or $\frac{5}{1\,4} \times \frac{2\,4\,0}{1} = \frac{5 \times 1\,2\,0}{7} = 85\frac{5}{7}d.$ *Ans.*

Let it be required to reduce $85\frac{5}{7}d.$ to the denomination of £.:—

Pence are reduced to £. by dividing successively by 12 and 20, or multiplying successively by $\frac{1}{1\,2}$ and $\frac{1}{2\,0}$. Accordingly, $\frac{6\,\frac{5}{7}\,0}{1} \times \frac{1}{1\,2} \times \frac{1}{2\,0} = \frac{5\,0}{7 \times 2\,4\,0} = £\frac{5}{1\,6\,8}$. *Ans.*

If it be required to convert $1\frac{1}{4}$ lb. into an equivalent quantity expressed in the denomination of cwt.:—

Lbs. become cwt. by division by 28 and 4; consequently $\frac{5}{4} \times \frac{1}{2\,8} \times \frac{1}{4} = \frac{5}{4\,4\,8}$ cwt. *Ans.*

Suppose we wish to express $\frac{5}{1\,4}$ of a guinea as an equivalent fraction of a £.:—

Guineas are reduced to £. by first multiplying by 21 and then dividing by 20; or by multiplying by $\frac{2\,1}{2\,0}$, for each guinea is $\frac{2\,1}{2\,0}$ of a £. Accordingly,

$$\frac{5}{1\,4} \times \frac{2\,1}{2\,0} = \frac{1 \times 3}{2 \times 4} = £\frac{3}{8}.$$

It is manifest from the above examples, that if the pupil has become familiar with fractional multiplication and division, the teacher will find little difficulty in enabling him to apply to concrete fractional quantities the principles of the common Rule of Reduction. Let the pupil be prompted to consider, with reference to a fractional part of a denomination, how he would reduce an integer of the same denomination; and it only remains for him to attend to the peculiar form in which multiplication or division is effected with fractions. In some books of Arithmetic, the Reduction of Concrete Fractions precedes the consideration of fractional multiplication. This is not a natural nor an expedient arrangement.

When the first 18 Exercises of Section 12th are performed, the following illustration may be entered upon.

When one quantity is proposed in several denominations, to be reduced to one equivalent higher denomination, either of two methods may be adopted.

Let it be proposed to express 12*s.* $10\frac{1}{2}d.$ in the denomination of £.:—

1*st Method.* 12*s.* $10\frac{1}{2}d.$ = 309 *hf. d.*; and each

halfpenny is the 480th of £1.; therefore, $12s.\ 10\frac{1}{2}d. = \frac{3}{4}\frac{0}{8}\frac{2}{0}$ of £1. $= £\frac{1}{1}\frac{9}{8}\frac{3}{0}$. *Ans.*

2nd Method. As each penny is $\frac{1}{20}s.$, therefore $10\frac{1}{2}d. = \frac{10\frac{1}{2}}{12} = \frac{8}{8}\frac{1}{4} = \frac{7}{8}s.$; also $12s.\ 10\frac{1}{2}d, = 12\frac{7}{8}s.$ Moreover each shilling is $£\frac{1}{2\,0}$, therefore

$$12\frac{7}{8}s. = \frac{12\frac{7}{8}}{20} = £\frac{1}{1}\frac{9}{8}\frac{3}{0}. \textit{ Ans.}$$

The latter method will be preferred by a practised arithmetician. But both may be required from the pupil in working Exercises 19 to 27.

When the integer values equivalent to given concrete fractions are required, the process is just the same as for the elementary Rule of Compound Division. Thus, $£\frac{5}{6} = £5 \div 6 = 16s.\ 8d.$; so also $5\frac{4}{7}$ hours = 5 ho. + 4 ho. ÷ 7 = 5 ho. 34 min. $17\frac{1}{7}$ sec.

Accordingly, Ex. 28 to 37 will be found exceedingly easy. And an inspection of the subjoined examples will be a sufficiently instructive preparation for the performance of the remaining Exercises of Section 12th.

Let it be required to find the sum and the difference of $1\frac{3}{5}$ qr. and $\frac{3}{1\,0}$ cwt. :—

$1\frac{3}{5}$ qr. = 1 qr. + 8 qrs. ÷ 9 = 1 qr. 24 lb. $14\frac{2}{5}$ oz.
$\frac{3}{1\,0}$ cwt. = 3 cwt. ÷ 10 = 1 5 $9\frac{3}{5}$

Ans.				
Sum	3	2	$7\frac{3}{4}\frac{7}{5}$
Diff.	0	19	$4\frac{2}{4}\frac{8}{5}$

Multiply £3. 16*s*. $5\frac{1}{2}d.$ by $11\frac{3}{5}$;—

$11\frac{3}{5}$

5)7 12 $11\frac{1}{5}$ = *twice* the given sum.

1 10 $7\frac{1}{4}\frac{1}{5}$ = $\frac{1}{2}$ of the given sum.
42 1 $1\frac{1}{5}$ = 11 times ditto.

£43 11 $8\frac{2}{1}\frac{2}{5}$ *Ans.*

Find the 8th part of 3 ro. 21 po. 26 sq. yds. 3¾ sq. ft.

8)141 po. 26 sq. yds. 3 sq. ft. 41¼ sq. in.

17 22 1 $117\frac{9}{14}$ *Ans.*

In this last example, the remainder 5 po. is multiplied by 30¼, and 26 yds. are added; then division by 8 leaves a remainder of 1¼ yd., which multiplied by 9 and 3 being added, the division by 8 leaves a remainder of 6¼ ft. &c.

How often is 3⅓ sec. contained in $\frac{4}{13}$ hour?

$\frac{4}{13}$ ho. × $\frac{60}{1}$ × $\frac{60}{1}$ = $\frac{14400}{13}$ sec., which divided by $\frac{39}{8}$ sec., or multiplied by $\frac{8}{39}$, gives $\frac{4\,0\,5\,0}{1\,3}$ = $311\frac{7}{13}$ times. *Ans.*

In concluding our observations on Vulgar Fractions, we may remark, that though they may appear, under a superficial view, to have little practical utility, they really are of great service in commercial affairs. A knowledge of them is a key to the general principles of calculation. It is recommended, therefore, that the pupil should be well practised in fractional computation. The Promiscuous Exercises in Section 13, will furnish a good test of his acquaintance with this subject.

EXERCISES.

12. *Reduction, &c., of Concrete Fractions.*

1. Express $£\frac{4}{19}$ in the denomination of shillings.
2. ..... $£\frac{9}{32}$ ..... pence.
3. ..... $£\frac{5}{17}$ ..... farthings.
4. ..... $\frac{5}{168}$ of a guinea ..... shillings.
5. ..... $\frac{7}{80}$ of a crown ..... half pence.
6. ..... $\frac{1}{2}$ of a farthing ..... shillings.
7. ..... $2\frac{1}{2}d.$ ..... £s.
8. ..... $\frac{3}{4}$ of a penny ..... guineas.
9. ..... $\frac{3}{80}$ of a cwt. ..... pounds.
10. ..... $\frac{9}{10}$ of a cwt. ..... ounces.
11. ..... $\frac{4}{17}$ of an ounce ..... tons.

Answers. 1. $4\frac{4}{19}s.$; 2. $27\frac{9}{19}d.$; 3. $282\frac{16}{17}q.$;
4. $\frac{5}{6}s.$, 5. $\frac{3}{8}\frac{3}{7}\frac{0}{8}hf.\ d.$; 6. $\frac{1}{84}s.$; 7. $£\frac{1}{56}.$;
8. $\frac{1}{378}gu.$; 9. $\frac{2}{84}\frac{2}{5}lb.$; 10. $1612\frac{4}{5}oz.$; 11. $\frac{1}{15680}t.$;

12. Express $\frac{3}{11}$ of a yard ..... nails.
13. ..... $\frac{9}{1000}$ of a mile..... inches.
14. ..... $6\frac{1}{3}$ inches ..... furlongs.
15. ..... $\frac{6}{11}$ ..... guineas.
16. ..... $\frac{5}{8}$ of a guinea ..... £s.
17. ..... $\frac{1}{144}$ of an acre ..... square yds.
18. ..... $\frac{1}{11}$ of an Eng. ell ..... yards.

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19. ..... $7s.\ 6\frac{1}{2}d.$ ..... £s.
20. ..... 2 bu. 3 pks. ..... quarters.
21. ..... 3 hours 15 min. ..... days.
22. ..... £4. $10s.\ 3\frac{1}{4}d.$ ..... £s.
23. ..... 5 cwt. 3 qrs. 21 lbs. ..... cwt.
24. ..... £1. $0s.\ 9\frac{3}{4}d.$ ..... guineas.
25. ..... 1 yd. $2\frac{1}{2}$ na. ..... Eng. ells.
26. ..... £73. $19s.\ 0\frac{1}{2}d.\ \frac{3}{7}q.$ ..... £s.
27. ..... 13 ho. 52 min. $50\frac{5}{11}$ sec. . . days of 12 hrs.

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28. Extend in integers the value of $£\frac{3}{8}$; 29. $£\frac{1}{320}$;
30. £. $\frac{1}{3}\frac{1}{4}$; 31. $\frac{5}{8}s.$ 32. $\frac{1}{8}$ guin.; 33. $£97\frac{9}{11}$;
34. $\frac{1}{3}\frac{1}{8}$ of $3s.\ 6d.$; 35. $\frac{5}{17}$ lb. troy; 36. $\frac{7}{11}$ ton;
37. $\frac{9}{11}$ of half a cwt.

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38. Add $£5\frac{1}{8}. + 5\frac{1}{8}s. + 5\frac{5}{8}d.$
39. Add $\frac{5}{11}$ ac. + $15\frac{1}{2}$ sq. ft. + $1\frac{9}{10}$ sq. yd. + $3\frac{3}{4}\frac{3}{4}$ ro.

Answers. 12. $3\frac{9}{11}na.$; 13. $570\frac{6}{25}in.$; 14. $\frac{1}{1320}fur.$;
15. $\frac{4}{77}gu.$; 16. $£\frac{7}{11}.$; 17. $33\frac{1}{6}sq.\ yd.$; 18. $1\frac{3}{32}yd.$;
19. $£\frac{1}{4}\frac{8}{10}.$; 20. $\frac{1}{12}qr.$; 21. $\frac{1}{5}\frac{3}{6}da.$; 22. $£4\frac{3}{4}\frac{3}{4}$;
23. $5\frac{1}{16}\frac{5}{6}cwt.$; 24. $\frac{1}{11}\frac{1}{2}gu.$; 25. $\frac{3}{4}\frac{7}{0}E.\ e.$;
26. $£73\frac{3}{8}\frac{9}{1}$; 27. $1\frac{3}{8}\frac{3}{1}\frac{1}{8}da.$; 28. $7s.\ 6d.$;
29. $8d.$; 30. $11s.\ 2\frac{1}{4}d.\ \frac{3}{5}q.$; 31. $6\frac{1}{2}d.\ \frac{3}{5}q.$;
32. $13s.\ 1\frac{1}{3}d.$; 33. £97. $18s.\ 2\frac{9}{11}d.$; 34. $1s.\ 10\frac{1}{2}d.$;
35. $9oz.\ 3dwt.\ 12\frac{1}{17}grs.$; 36. $9cwt.\ 1qr.\ 9\frac{1}{3}lb.$;
37. $1qr.\ 15\frac{1}{13}lb.$; 38. £5. 17s. $1\frac{1}{2}$d. $\frac{3}{5}$q.
39. $1ac.\ 1ro.\ 2po.\ 2yd.\ 6ft.\ 50\frac{3}{4}in.$

40. Deduct $\frac{7}{11}$ cwt. from $3\frac{1}{1\,4}$ qrs.
41. Subtract $\frac{7}{16}$ of 7*s.* 6*d.* from £$\frac{3}{8}$.
42. What is the difference between $\frac{5}{14}$ yd. and $\frac{2}{16}$ Eng. ell?
43. £25. 3*s.* 6*d.* × $8\frac{1}{2}$. 44. £5. 12*s.* $10\frac{1}{2}d.$ × $15\frac{1}{2}$.
45. £21. 10*s.* $9\frac{1}{3}d.$ × $2\frac{2}{3}$. 46. £1. 4*s.* $7\frac{3}{11}d.$ ÷ 14.
47. Find the 7th part of 16 ac. 3 ro. $32\frac{1}{4}$ po.
48. 50 cwt. 3 qrs. $23\frac{1}{4}$ lbs. ÷ $4\frac{2}{3}$.
49. Divide £15. $16\frac{1}{4}\frac{1}{5}$*s.* by 12*s.* $5\frac{1}{4}d.$
50. Divide 2 yds. 1 ft. $10\frac{1}{2}$ in. by 4 yd. 1 ft. $5\frac{1}{2}$ in.
51. How often is $\frac{1}{1\,2}$ of a lineal yard contained in $\frac{5}{8}$ of a pole?

Answers. 40. $12\frac{1}{1}\frac{0}{1}lb.$; 41. 4*s.* $2\frac{1}{2}d.$ $\frac{1}{2}q.$;
42. 1*qr.* 1*na.* $1\frac{3}{1}\frac{3}{2}in.$; 43. £206. 8*s.* $8\frac{1}{3}d.$;
44. £88. 13*s.* $5\frac{1}{1\,4}d.$; 45. £57. 8*s.* $10\frac{1}{3}\frac{2}{5}d.$;
46. 1*s.* $9\frac{1}{1\,1}d.$ 47. 2*ac.* 1*ro.* $27\frac{1}{2}\frac{1}{2}po.$; 48. 11*cwt.* 2*lb.*;
49. $25\frac{1}{3}$ *times*; 50. $\frac{7}{1\,4}$ *of once*; 51. $5\frac{1}{7}$ *ti.*

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13. Promiscuous Exercises.

1. What is the sum of $\frac{4}{7}$ and $\frac{7}{5}$? *Ans.* $2\frac{4}{3}\frac{3}{5}$.
2. What is the difference of $\frac{7}{1\,0}$ and $\frac{5}{8}$? *Ans.* $\frac{2}{1\,5}$.
3. I have spent $\frac{5}{1\,2} + \frac{1}{3}$ of my money; what part of the original sum remains? *Ans.* $\frac{1}{3}\frac{7}{6}$.
4. How many 30ths are in $\frac{4}{5}$? *Ans.* $16\frac{2}{3}$.
5. What cost $\frac{5}{6}$ of a yd. at 2*s.* $10\frac{1}{2}d.$ per yd.? *Ans.* 2*s.* $4\frac{3}{4}d.$
6. Add 4 *fourths* of $33\frac{1}{3}$ to 12 *thirds* of $27\frac{3}{1\,6}$. *Ans.* $141\frac{1}{2}$.
7. How often is $52\frac{1}{2}$ contained in 21? *Ans.* $\frac{2}{5}$ of *once.*
8. How many 40ths of a cwt. must be added to 40 cwt. to make 41 cwt.? *Ans.* 40.
9. Reduce 5915410 square inches to roods, &c. *Ans.* 3*ro.* 30*po.* 26*yds.* 7*ft.* 106*in.*

10. If $\frac{4}{5}\frac{1}{3}$ of my money is 4*s.* $3\frac{1}{2}d.$; required the whole. *Ans.* 6*s.* $6\frac{3}{4}d.$

11. Multiply $3\frac{1}{2}$ *tenths* by $\frac{1}{1}\frac{1}{2}$ of 3. *Ans.* $\frac{2}{6}\frac{1}{8}$.

12. How many 12ths of a £. are in 4*s.* 6*d.*? *Ans.* $2\frac{7}{1}\frac{}{0}$.

13. How many 8ths of a furlong are in $\frac{1}{7}$ of a mile? *Ans.* $9\frac{1}{7}$.

14. How many 27ths of a square yd. are in $\frac{5}{7}$ of an acre? *Ans.* $37337\frac{1}{7}$.

15. Divide $3\frac{1}{3}$ *twelfths* by the 8th part of 6. *Ans.* $\frac{2}{3}$.

16. How many 25ths of a £. must be subtracted from £32. to leave £26.? *Ans.* 150.

17. A. gets £2. 16*s.*, B. gets $\frac{3}{1}\frac{}{0}$ of A., and C. gets $2\frac{1}{2}$ times B. What sum is divided among them? *Ans.* £5. 14*s.* $9\frac{3}{4}d.$

18. What fraction of a £. subtracted from $\frac{3}{2}\frac{}{0}$ of a shilling leaves $\frac{3}{2}\frac{}{0}$ of a penny? *Ans.* $£\frac{1}{2}\frac{1}{0}\frac{}{0}$.

19. If $\frac{1}{8}$ of 7 times the 6th of my farm measures 140 acres; what is its whole extent? *Ans.* 180*ac.*

20. If $\frac{1}{8}$ of a lb. of tea cost $1\frac{7}{1}\frac{}{1}$ shill.; how many lbs. may be had for $2\frac{7}{1}$ shill.? *Ans* $\frac{1}{1}\frac{7}{2}\frac{5}{0}lb.$

21. If $15\frac{2}{3}$ grains of silver be worth $2\frac{1}{4}d.$; what is the worth of $\frac{5}{7}$ dwt.? *Ans.* $1\frac{3}{4}\frac{}{7}d.$

22. If $10\frac{7}{1}\frac{}{7}$ yds. cost 29*s.* 3*d.*; how many yards cost 12*s.* 9*d.*? *Ans.* $4\frac{7}{1}\frac{}{3}$.

23. Whether is $\frac{1}{1}\frac{}{3}$ of a quarter of corn or $\frac{5}{8}$ of a bushel the greater quantity? Express the difference in pecks. *Ans.* Former, by $\frac{1}{8}pk.$

24. What is the difference, in inches, between $\frac{2}{3}$ of a yard and $\frac{2}{3}$ of a foot? *Ans.* $5\frac{1}{3}in.$

25. Can a father, in dividing his property among his three sons, give $\frac{1}{2}$ of it to the first, $\frac{2}{3}$ to the second, and $\frac{3}{4}$ to the third?

Ans. No. Property too little by $\frac{1}{4}\frac{}{0}$ of itself.

26. The quarter of $\frac{2}{3}$ of my money is the same as the eighth of 2*s.* 4*d.* How much have I? *Ans* 1*s.* 9*d.*

27. After I had gone $\frac{3}{4}$ of $\frac{1}{2}$ of twice my journey, I had $10\frac{1}{2}$ miles further to travel. Required the whole journey? Ans. 12mi.

28. If $\frac{4}{7}$ of the half of a number exceed $\frac{2}{7}$ of the fourth of that number by 20; what is the number? *Ans.* 112.

29. I spent $\frac{1}{3}$ of my money yesterday, and $\frac{1}{4}$ of the remainder to day; and I have thereby left myself 15*s.* 4½*d.* How much had I at first? *Ans.* £2. 2*s.* 8½*d.*

30. The sum of £37. 12*s.* 6*d.* was diminished by successive subtractions to £15. 1*s.* and £4. 6*s.* What fractional parts of the original sum were successively taken away? *Ans.* $\frac{3}{5}$ and $\frac{1}{7}$.

31. One-third of my money exceeds one-seventh of the remainder by 12*s.* 6*d.* How much have I?

Ans. £2. 12*s.* 6*d.*

32. A. and B. are travelling to the same inn; and when A.'s distance from it is $6\frac{2}{3}$ miles, B.'s is 5 miles. If B.'s rate is $3\frac{1}{8}$ miles an hour, what must A.'s be, that they may arrive at the same time? *Ans.* $4\frac{1}{6}$*mi.*

33. If my present age were a fifth of itself greater, I should be $13\frac{1}{2}$ years old. Required my age. *Ans.* $11\frac{1}{4}$.

34. If my present age were a fifth of itself less, I should be 14 years old. Required my age. *Ans.* $17\frac{1}{2}$.

35. One seventh of my money, added to three-fourths of thrice the remainder, would amount to 3*s.* 7½*d.* How much have I? *Ans.* 1*s.* 9*d.*

36. One horse can clear 9 miles in 32 minutes, and another horse $7\frac{1}{2}$ miles in 28 minutes. Which is the swifter? *Ans.* Former to latter as 21 : 20.

37. From a quantity of sugar there was taken away, first $\frac{1}{7}$ of it, and then $\frac{1}{2}$ the remainder, thereby leaving $\frac{1}{2}$ cwt. Find the original quantity. *Ans.* $2\frac{1}{8}$*cwt.*

38. If the value of $5\frac{1}{2}$ oz. of coinage silver is 29*s.* 4*d.*, and that of $3\frac{1}{4}$ oz. of pure silver is 20*s.* $9\frac{3}{7}$*d.*; how many ozs. of pure silver are equivalent to 60 ozs. of the coinage metal? *Ans.* $55\frac{1}{2}$*ozs.*

39. If 7 men could perform $1\frac{1}{3}$ of a work in $3\frac{6}{11}$ days, working $8\frac{1}{4}$ hours a day; how many hours a day should 5 men be engaged, that the whole work may be done in $4\frac{1}{2}$ days? *Ans.* $10\frac{10}{66}$*hrs*

40. If $\frac{1}{7}$ of a lb. of sugar, or $\frac{1}{8}$ of an oz. of tea, cost

$3\frac{3}{4}d.$; what is the cost of a parcel containing 2lbs. $3\frac{2}{15}$ oz. of tea, and 3 lbs. $6\frac{1}{4}$ oz. of sugar? *Ans.* $16s.\ 7\frac{1}{2}d.$

41. If I spend, in 3 years, $2\frac{1}{7}$ of my annual income; what part of nine months' income do I save every six months? *Ans.* $\frac{8}{21}$.

42. A. can perform, in 36 days, a work which would occupy A. and B. together 16 days. In what time would the work be finished by B. alone? *Ans.* $28\frac{4}{5}da.$

43. If 5 horses can eat 3 trusses of hay in 12 hours, and 2 oxen can do the same in 54 hours; in what time can 5 horses and 2 oxen together do it? *Ans.* $9\frac{9}{11}hrs.$

44. A farmer employed 20 reapers, who cut down his crop in 14 days. Now, 7 of these persons could have reaped the whole in 45 days, and 5 others could have done the same in 63 days. In what time could the 8 persons remaining have cut down half the crop? *Ans.* $15da.$

45. If $\frac{1}{7}$ of an oz. of tea be worth $\frac{3}{8}$ of a lb. of sugar, and $\frac{1}{5}$ of a cwt. of sugar worth $\frac{3}{4}$ of a gallon of rum, and $\frac{1}{11}$ of a quart of rum worth $1s.\ 3\frac{1}{4}d.$ less than a lb. of tea; what is the worth of $6\frac{1}{2}$ lbs. of sugar?

Ans. $2s.\ 11\frac{1}{4}d.$

XV. DECIMAL FRACTIONS.

Conversion of Vulgar into Decimal Fractions.

We have shown that any Vulgar Fraction may be converted into an equivalent one having any proposed Denominator. We may, therefore, convert any given fraction into one that shall have for its denominator 10, or 100, or 1000, &c.; that is, into a fraction having for its denominator 1 with a cipher or ciphers annexed. Such reduction produces what are called *Decimal Fractions.*

Definition.—DECIMAL FRACTIONS are such as are expressed in tenths, hundredths, thousandths, &c. that their notation may correspond to the notation of integers.

Examp. What numerator must $\frac{17}{32}$ assume, that its denominator may be converted into 10, into 100, into 1000, into 10000, into 100000?

17 units = 170 tenths;
17 units $\div$ 32 = $5\frac{5}{16}$ tenths. *Ans.*

17 units = 1700 hundredths;
17 units $\div$ 32 = $53\frac{1}{8}$ hundredths. *Ans.* &c.

Now, it will appear that an exact numerator will not be obtained till we can make the denominator 100000.

17 units = 1700000 hundred-thousandths;
17 units $\div$ 32 = 53125 hundred-thousandths. *Ans.*

It appears, then, that to convert a Vulgar Fraction into a Decimal, we should annex one cipher after another to the given numerator, to make it admit of division by the given denominator, and the quotient figure arising from the annexation of the first cipher, will denote so many tenths; from the second, so many hundredths; from the third, so many thousandths; and so forth. Now, we may stop the division when we please. If, for instance, we stop before dividing thousandths, we shall obtain, as the decimal form of $\frac{17}{32}$, the mixed fraction $53\frac{1}{8}$ *hundredths*, but it is generally expedient to carry out the division to its exact limit, when that is attainable within eight or nine decimal places.

It most frequently happens, however, that the division will never reach an exact termination.* Thus, let it be required to convert $\frac{5}{13}$ into the decimal form:—

The division, as will be seen in the margin, yields a numerator which will consist of a perpetual recurrence of the figures 384615; we cannot, therefore, obtain a pure decimal; but this imperfection is of no

13)5000000000
38461538

* It never reaches an exact termination unless the divisor can be divided down to unity by 5 or 2.

consequence in the practical applications of Arithmetic, as a portion of the interminate numerator will be found a sufficiently accurate expression of the value of $\frac{5}{18}$. If we take the first four figures, $\frac{8846}{10000}$, we neglect only $\frac{2}{18}$ of the *ten thousandth* part of a unit. If we take the first six figures, $\frac{884615}{1000000}$, we neglect only $\frac{5}{18}$ of the *millionth* part of a unit, or less than the *two-millionth*.

It may now be observed, that if the first or left-hand figure of the numerator of a decimal fraction be always the place of *tenths*, the denominator will be thereby determined to consist of 1, with a cipher for each figure of the numerator. Accordingly, $\frac{6}{1000}$, as it consists of $\frac{0}{10} + \frac{0}{100} + \frac{6}{1000}$, will be represented by $\frac{006}{1000}$, in order that the number of ciphers in the denominator may equal the number of figures in the numerator. As this form of expression has been agreed upon, we do not require to write the denominator of a decimal fraction; only, we must have some sign upon the numerator to prevent its being mistaken for an integral quantity. The sign customarily employed for this purpose, is an elevated point preceding the place of tenths; thus, ·5 denotes $\frac{5}{10}$, and ·025 denotes $\frac{025}{1000}$ or $\frac{25}{1000}$.

Accordingly, in converting Vulgar into Decimal Fractions, the correct form of operation is that exemplified in the margin, by the reduction of $\frac{1}{16}$ to the equivalent decimal, 625 *ten-thousandths*. In this particular example, the dividend may be named either 1 *unit*, or 10 *tenths*, or 100 *hundredths*, &c., and a cipher must occupy the quotient place of tenths, to give the following figure its proper local indication as hundredths.

16)1·0000
·0625

Ciphers on the left, in a decimal numerator, affect its value; on the right they do not. Thus, ·05 is not = ·5, because ·05 = $\frac{5}{100}$, whereas ·5 = $\frac{5}{10}$; but ·5 is = ·50, because $\frac{5}{10} = \frac{50}{100}$.

When, for convenience, we retain only an approximate value of a decimal, and the first two of the figures *which we reject* are greater than 50, we should increase

by 1 the last of the figures which we retain; because, then the excess above accuracy would have been smaller than the defect below accuracy would have been. Thus, as an approximate value for ·46274, it is evident that ·463 is nearer the truth than ·462 would be.

Before concluding this section, we shall exemplify a convenient method of extending certain decimal quotients, without the trouble of division.

Suppose we are required to convert $\frac{1}{17}$ to its decimal equivalent:—

Dividing 1 by 17, we obtain $\cdot 0588235294\frac{2}{17}$ as the value of $\frac{1}{17}$; therefore $\frac{2}{17} = \cdot 1176470588\frac{4}{17}$, obtained by doubling the quotient; and hence, by substitution of decimals, we have $\frac{1}{17} = \cdot 0588235294117647$, &c.

It is, however, rarely necessary to employ in calculation such a degree of decimal precision as this method so rapidly attains; and when absolute accuracy is required, fractions of the vulgar form are generally preferable to large decimals.

EXERCISES.

1. *Conversion of* Vulgar *into* Decimal *Fractions.*

Reduce to *tenths*, *hundredths*, &c. (1) $\frac{4}{5}$; (2) $\frac{1}{2}$; (3) $\frac{3}{5}$; (4) $\frac{3}{4}$; (5) $\frac{5}{8}$; (6) $\frac{1}{20}$; (7) $\frac{11}{16}$; (8) $\frac{201}{400}$; (9) $\frac{1}{2000}$.

Represent by decimal notation (10) $\frac{1}{5}$; (11) $\frac{3}{5}$; (12) $\frac{9}{1000}$; (13) $\frac{9}{50}$; (14) $\frac{1}{16}$; (15) $46\frac{8}{15}$; (16) $\frac{49}{125}$; (17) $\frac{1}{625}$; (18) $\frac{7}{41}$; (19) $\frac{8}{625}$; (20) $\frac{17}{25}$.

Reduce (21) $\frac{3}{8}$ to thousandths; (22) $\frac{33}{80}$ to ten-thousandths; (23) $\frac{1}{4}$ to tenths; (24) $\frac{7}{8}$ to hundredths; (25) $\frac{21}{250}$ to tenths, to hundredths, and to thousandths; (26) $\frac{4}{7}$ to thousandths; (27) $\frac{1}{17}$ to ten-thousandths.

Answers. 1. $\frac{8}{10}$ or ·8; 2. $\frac{5}{10}$ or ·5; 3. $\frac{6}{10}$ or ·6; 4. $\frac{7}{10} + \frac{5}{100}$, or ·75; 5. $\frac{6}{10} + \frac{2}{100} + \frac{5}{1000}$, or ·625; 6. $\frac{0}{10} + \frac{5}{100}$, or ·05; 7. ·9375; 8. ·5075; 9. ·0005; 10. ·444, &c.; 11. ·8333, &c.; 12. ·82727, &c.; 13. ·243243, &c.; 14. ·0555, &c.; 15. 46·375; 16. 3·2666, &c.; 17. ·00108108, &c.; 18. ·1707317, &c.; 19. ·00548—; 20. ·2881356—; 21. $\frac{625}{1000}$ or ·625; 22. ·6625; 23. $2\frac{1}{2}$; 24. $\cdot 87\frac{1}{2}$; *25.* $\cdot 8\frac{12}{13}$, $\cdot 87\frac{1}{8}$, ·876; 26. $\cdot 571\frac{3}{7}$; 27. $\cdot 0001\frac{3}{7}$.

Conversion of Decimal into Vulgar Fractions.

The pupil having worked the exercises in Sect. 1st, may then learn to reduce decimal fractions to the form of vulgar fractions. This, of course, may be accomplished by depriving the given decimal of its point, and writing it over its understood denominator. Let the fraction then be reduced, if possible, to lower terms.

Examp. Reduce ·825, and 24·04, to the vulgar form.

$\frac{825}{1000} = \frac{33}{40}$ *Ans.* $\qquad 24\frac{4}{100} = 24\frac{1}{25}$ *Ans.*

When the pupil has arrived at the 13th Exercise of Section 2nd, he should be made acquainted with the usual notation of *Recurring* or *Repeating* Decimals, that is, of Decimals in which one or more figures perpetually recur, as ·333, &c., ·2323, &c., ·05374374, &c.

Interminate Decimals of this form are usually indicated by head points. If a single figure recur, it is written once, with a point over it; and if a period of figures recur, it is called a circulating decimal, and is denoted by two points over the first and last figures of the period, respectively. Thus, ·444, &c., is written $\cdot\dot{4}$; also, ·62727, &c., is written $\cdot\dot{6}2\dot{7}$; and 7·630630, &c., is written $7\cdot\dot{6}3\dot{0}$.

Recurring Decimals are convertible into finite fractions, by writing 9, 99, &c., instead of 10, 100, &c., as the denominator. Thus, $\cdot\dot{7}\dot{6} = \frac{76}{99}$. A demonstrative illustration of this may be given in the following manner:—

Since $\quad 7600 = 76 \times 99 + 76,$
therefore $\frac{7600}{99} = 76 \qquad + \frac{76}{99}.$

Hence, any number consisting of two digits, if divided by 99, will yield the same digits for the first two decimal figures of the quotient, and the same digits for a remainder; so that a continuation of the division will

produce a perpetual recurrence of the same two digits in the quotient.

In like manner it may be shown, that any other number, if divided by as many *nines* as it contains digits, will yield these digits as a decimal quotient, together with the same digits as a remainder.*

When decimal figures precede the recurring part, a complex fraction will arise, which is treated as follows:—

Examp. Convert $1{\cdot}04\dot{8}$, and $\cdot3\dot{0}\dot{6}$ to the vulgar form.

$$1{\cdot}04\tfrac{8}{9} = 1\tfrac{48}{100} = 1\tfrac{44}{900} = 1\tfrac{11}{225}\text{—}\textit{Ans.}$$

$$\cdot3\tfrac{06}{99} = \frac{3\frac{2}{33}}{10} = \tfrac{101}{330}\ \textit{Ans.}$$

EXERCISES.

2. *Conversion of* Decimal *into* Vulgar *Fractions.*

Reduce to the vulgar form (1) ·7; (2) ·36; (3) ·45; (4) ·325; (5) ·409; (6) 21·35; (7) 12·375; (8) ·03125; (9) ·0011; (10) $\cdot142\frac{8}{7}$; (11) $\cdot58\frac{1}{3}$; (12) $3{\cdot}227\frac{1}{2}$; (13) $\cdot\dot{7}$; (14) $\cdot\dot{6}$; (15) $\cdot\dot{1}\dot{2}$; (16) $\cdot5\dot{8}\dot{5}$; (17) $\cdot4\dot{8}\dot{1}$; (18) $\cdot2\dot{3}$; (19) $\cdot6\dot{8}\dot{1}$; (20) $\cdot7\dot{0}0\dot{5}$; (21) $4{\cdot}1\dot{4}\dot{4}$; (22) $25{\cdot}00123$; (23) $\cdot00047\dot{3}\dot{4}\dot{8}$.

Answers. 1. $\frac{7}{10}$; 2. $\frac{9}{25}$; 3. $\frac{9}{20}$; 4. $\frac{13}{40}$; 5. $\frac{409}{1000}$; 6. $21\frac{7}{20}$; 7. $12\frac{3}{8}$; 8. $\frac{1}{32}$; 9. $\frac{11}{1000}$; 10. $\frac{1}{7}$; 11. $\frac{7}{12}$; 12. $3\frac{91}{400}$; 13. $\frac{7}{9}$; 14. $\frac{2}{3}$; 15. $\frac{4}{33}$; 16. $\frac{65}{111}$; 17. $\frac{13}{27}$; 18. $\frac{7}{30}$; 19. $\frac{15}{24}$; 20. $\frac{1867}{1980}$; 21. $4\frac{16}{111}$; 22. $25\frac{41}{3330}$; 23. $\frac{1}{2112}$.

* With respect to recurring decimals, it may be said generally, that they are not so convenient in computation as their corresponding vulgar fractions, unless we are permitted to use them approximately, like other decimals.

"*Complete* operations with circulating decimals," says De Morgan, "are *ignes fatui* which have led many an arithmetical writer astray. . . . . Attempts to compute with interminable fractions are things on which real business will never waste a thought."—*Notices of Arithmetical Books*, pp. 69, 75.

Addition and Subtraction of Decimals.

Addition and Subtraction of Decimals are similar to those of Integers; but as the constant commencement of integers is at the place of units, or at the right hand, whereas the constant commencement of decimals is at the place of tenths, or at the left hand, we shall always have a full column on the left in decimals, whether the other columns be partial or not; whereas the reverse obtains with integers.

Examp. Required the sum and the difference of ·46 and ·366.

	·46		·46
	·366		·366
Sum	·826	Diff.	·094

In these examples, the principle of arrangement is that of placing all the tenths in one column, all the hundredths in another, &c. This will make all the decimal points fall into one column. Vacant places on the right may be conceived as being supplied with ciphers, because ciphers on the right of a decimal do not affect its value. The addition, or subtraction, then proceeds just as with integers; because in decimal fractions, as in whole numbers, the local values increase from right to left by a tenfold proportion.

Examp. Add, decimally, $2\frac{5}{13}$, $27\frac{7}{22}$, $5\frac{12}{17}$, and $\frac{3}{8}$.

2·4167	2·4166667
27·3182	27·3181818
5·3243	5·3243243
·375	·375
35·4342	35·4341728

We have reduced the vulgar fractions to approximate decimals, with two different degrees of accuracy. The first answer is about three 100000th parts of a unit above the truth; the second about a ten-millionth part of a unit below it.

It is, however, generally expedient, in finding approxi-

mate answers, to make the operation extend one or two figures beyond that at which we mean the answer to terminate.

Examp. Subtract $\cdot\dot{4}2\dot{6}$ from $20\frac{1}{237}$, securing five decimal places.

$$20\cdot0344827 - \cdot4262626 = 19\cdot60822 + \quad \textit{Ans.}$$

The teacher may easily extend the number of Exercises in Addition and Subtraction of Decimals by selecting from Sections 8 and 9 of Vulgar Fractions, and requiring the pupil, first, to reduce the given fractions to decimals; then, to add or subtract as is required; and lastly, to reduce the given answer to decimal form, and compare it with the answer previously obtained.

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3. *Addition of Decimals.*

1. Add $25\cdot6 + 36\cdot96 + 27\cdot8 + 38\cdot583 + 46\cdot072$. *Ans.* $175\cdot015$.
2. Add $\cdot17 + \cdot538 + \cdot25 + \cdot9 + \cdot265$. *Ans.* $2\cdot123$.
3. Add $5\cdot65 + \cdot7 + \cdot365 + 33\cdot07 + 16\cdot5294 + 65$. *Ans.* $121\cdot3144$.
4. Convert into decimals, and add, $\frac{1}{8}$, $\frac{2}{5}$, $\frac{1}{6}\frac{2}{4}$, $\frac{2}{3}$, and $\frac{9}{3\cdot3}$. *Ans.* $1\cdot564603 -$.
5. Add, decimally, 3 hundredths, 9 tenths, 19 ten-thousandths, and 213 thousandths. *Ans.* $1\cdot1449$.
6. Add, decimally, $\frac{1}{4}\frac{2}{5} + \frac{5}{9} + \frac{1}{11} + \frac{2}{5\cdot5}$. *Ans.* $\cdot\dot{9}7\dot{1}$.
7. Add, $\cdot\dot{2}7\dot{6} + 17\cdot\dot{6}5\dot{4} + 3\cdot6 + \cdot\dot{2}6\dot{3}5$. *Ans.* $21\cdot8610147 +$.
8. Add, $1\cdot\dot{2}4\dot{6} + \frac{2}{7} + \cdot\dot{0}0\dot{5} + 1\frac{1}{11} + 17\cdot\dot{6}2\dot{2}0$. *Ans.* $20\cdot678732 -$.

4. *Subtraction of Decimals.*

1. Subtract $7\cdot37$ from $36\cdot46$. *Ans.* $29\cdot09$.
2. Subtract $\cdot3476$ from $\cdot9046$. *Ans.* $\cdot557$.
3. From $\cdot5632$ subtract $\cdot09763$. *Ans.* $\cdot46557$.
4. *From* $1\cdot0101$ subtract $\cdot101$ *Ans.* $\cdot9091$.

5. Subtract 5·332 from 53·32. *Ans.* 47·988.
6. Subtract ·80738 from 3·605. *Ans.* 2·79762.
7. Subtract ·365 from 365. *Ans.* 364·635.
8. From ·23̇ subtract ·1̇37̇. *Ans.* ·095186 +.
9. From ·0003̇ subtract 9 hundred-thousandths. *Ans.* ·000243.

Multiplication and Division of Decimals.

The 10th part of a number may be represented, by so placing the decimal point between two of its figures, as to make what before was *tens* to represent *units*, what was *units* to represent *tenths*, what was *tenths* to become *hundredths*, &c. This will evidently be effected by shifting the point from its place in the given number to the other side of the figure which precedes it. Thus, 7·6 ÷ 10 = ·76, that is, the 10th part of 7 *units* and 6 *tenths* is 7 *tenths* and 6 *hundredths*. So also 7 ÷ 10, or 7·0 ÷ 10 = ·70 or ·7.

In like manner, the 100th part of a number may be represented by shifting the decimal point two places to the left. And, universally, when a whole number, consisting of 1 with ciphers annexed, is used as a divisor, the quotient may be represented by merely shifting the decimal point of the dividend as many places to the left as there are ciphers in the divisor. Thus,

$$860 \div 100 = 860{\cdot}0 \div 100 = 8{\cdot}6;$$
$$45 \div 1000 = 45{\cdot}0 \div 1000 = {\cdot}045;$$
$$23{\cdot}47 \div 10000 = {\cdot}002347.$$

On the same principle, a number is multiplied by 10, or made 10 times as great in value, when its decimal point is transferred one place to the right; by 100, when two places to the right; and so on. Thus,

$$42 \times 10 = 42{\cdot}0 \times 10 = 420;$$
$$3{\cdot}0652 \times 100 = 306{\cdot}52;$$
$${\cdot}004 \times 10000 = {\cdot}00400 \times 10000 = 0040{\cdot}0 = 40.$$

Suppose, now, we are required to multiply 6·39 by 7·5; the multiplicand is $= \frac{600}{100} + \frac{39}{100}$, or $\frac{639}{100}$; similarly, the multiplier is $= \frac{75}{10}$. Therefore,

$$6{\cdot}39 \times 7{\cdot}5 = \frac{639}{100} \times \frac{75}{10} = \frac{639 \times 75}{1000} = \frac{47925}{1000} = 47{\cdot}925. \textit{ Ans.}$$

Hence, it will be evident that, in decimal multiplication, we may determine the product as if for whole numbers, and then divide it by 1 with as many ciphers annexed as there are decimals in both factors; or, which is the same thing, point off from the right of the product as many decimal places as are contained in both factors.

Examp. Multiply 49·325 by ·0365; also, 4·68 by ·0037.

	49·325	·003777
In the second of these examples, we have made allowance for the figures which are carried from the products of the repeating figure.	·0365	4·68
	246625	302
	295950	2267
	147975	15111
	1·8003625	·017680

Again, suppose it is required to divide 6·39 by 7·5:

$$\frac{6\,3\,9}{1\,0\,0} \div \frac{7\,5}{1\,0} = \frac{6\,3\,9\cdot0}{1\,0\,0} \div \frac{7\,5}{1\,0} = 63{\cdot}9 \div 75 = {\cdot}852.$$

Hence, to divide decimally, multiply the divisor by 1 with as many ciphers as will make an integral product; which is just removing the decimal point as many places to the right of the divisor as there are decimal places in that divisor. Remove the point in the dividend the same number of places to the right. Then divide as in whole numbers, and place a decimal point in the quotient, just when you are proceeding to include the tenths of the dividend in a partial dividend.

Examp. Divide 187·643 by 5·31, and ·015 by ·1847.

531)18764·300	1847)0150·0000
Ans. 35·338	*Ans.* ·0812

The number of Exercises in Multiplication and Division of Decimals may be extended, if necessary, by prescribing the answers in Section 6th to be multiplied by the given divisors, and the answers in Section 5th to be divided by the given multipliers.

Contracted Multiplication and Division of Decimals.

The following examples of approximation will teach the pupil how to obtain decimal products and quotients with any assigned degree of precision.

Examp. Multiply 2·369578 by 3·58796, preserving accuracy in the product as far as the sixth decimal.

Entire Process.		*Contracted Process.*
2·369578		23'6'9'5'7'8'
3·58796		358796
142	17468	142
2132	6202	2133
16587	046	16587
189566	24	189566
1184789	0	1184789
7108734		7108734
8·501951	08088	8·501951

To understand the contracting operation, observe, that as the number of decimal figures in the two factors is 11, and we wish to retain only 6 in the product, we cut off, by a trait or accent, 5 figures from the right of the multiplicand. We then multiply 23 by the right hand figure of the multiplier, taking in what would have been carried in the complete process; this gives the first partial product. We next mark off 236 to be similarly multiplied by 9, &c.; making all the right hand figures of the several products stand in one column. A right hand figure, besides being increased by car-

riage, should be further increased by 1, if the figure that should have stood on its right, in the complete process, is greater than, or very nearly equal to, 5.

When the number of figures to be marked off in the multiplicand is greater than the multiplicand contains, supply ciphers on the left, as in the annexed method of multiplying ·684 by ·639, to secure two places of decimals.

```
0'0'6'84
  639
-------
    1
    2
   41
-------
 ·44
```

Examp. Divide 46·271 by 6·0579, stopping the quotient at the 6th decimal.

The mode of abbreviation begins by making the divisor a whole number, containing *at least* as many figures as are required in the quotient, the dividend being made conformable, by shifting its decimal point. We then multiply the entire divisor by the first quotient figure, and subtract; then multiply all but the right hand figure of the divisor by the next quotient figure, &c., marking off the figures of the divisor, one by one from right to left, and neglecting those on the right of the accent, except in the employment of their carriage figures.

```
                6'0'5'7'9'0'0)46271000
                              -------
                Ans.          7·638125
                              42405300
                              --------
                               3865700
                               3634740
                              --------
                                230960
                                181737
                              -------
                                 49223
                                 48463
                              -------
                                   760
                                   606
                                  ---
                                   154
                                   121
                                  ---
                                    33
                                    30
                                   ---
```

EXERCISES.

5. *Multiplication of Decimals.*

1.	25·67	×	34·5	=	885·615.
2.	2·936	×	2·7	=	7·9272.
3.	43·68	×	·58	=	25·3344.
4.	29·345	×	28	=	821·66.
5.	·1634	×	4·5	=	·7353.
6.	·34697	×	·25	=	·0867425.
7.	1·056	×	·123	=	·129888.
8.	·03675	×	·00328	=	·00012054.
9.	5·8375	×	720	=	4203.
10.	·05483	×	100	=	5·483.

6. *Division of Decimals.*

1.	·7492	÷	6	=	·12486.
2.	3937·5	÷	12	=	328·125.
3.	42·3969	÷	3·5	=	12·1134.
4.	305·736	÷	1·63	=	187·568+.
5.	298·2	÷	·365	=	816·986+.
6.	46271	÷	·059	=	784254·24−.
7.	·0635	÷	·0048	=	13·22916.
8.	251	÷	1000	=	·251.
9.	365	÷	7296	=	·0500274+.
10.	·324	÷	36	=	·009.

7. *Reduction of Concrete Decimals.*

Find the integer values in (1) £26·375; (2) £·08746; (3) £·09625; (4) 2·35756 ac.; (5) ·08995 shill.; (6) 123·5695 cwt.; (7) ·00964 cwt.; (8) 5·336⅛ lbs. troy; (9) 24·6375 gall.; (10) ·2790156 cub. yd.; (11) ·00075⅝ days of 10½ hrs.

Answers. 1. £26. 7*s.* 6*d.*; 2. 7*s.* 5¾*d.* ·62*q.*;
3. 1*s.* 11·1*d.*; 4. 2*ac.* 1*ro.* 17·2*po.*; 5. 1·08*d.*;
6. 123*cwt.* 2*qrs.* 7·78*lbs*; 7. 1*lb.* 1·275*oz.*;
8. 5*lb.* 4*oz.* 16*grs.*; 9. 24*gal.* 2*qt.* 1·1*pt.*;
10. 7*ft.* 921¾*in. cub.*; 11. 28·602sec.

Express (12) 17*s.* 9*d.* in £.; (13) $10\frac{1}{2}d.$ in shill.; (14) £5. 6*s.* $7\frac{3}{4}d.$ in £.; (15) $2\frac{1}{2}d.$ in £.; (16) 3 qrs. 2 bu. 3 pks. in qrs.; (17) 14 cwt. 1 qr. 23 lbs. in tons.; (18) 6 ac. 3 ro. 31 po. $21\frac{1}{2}$yds. in roods; (19) $30\frac{1}{4}$ sec. in days; (20) 3·47*d.* in crowns.

(21.) Convert 36·4 lbs. of barley into lbs. of malt, at the rate of 39·926 lbs. of malt per bushel of 50·38 lbs. barley.

Answers. 12. £·8875.; 13. ·875*s.*; 14. £5·332292; 15. £·010416; 16. 3·34375*qrs.*; 17. ·722768*ton*; 18. 27·7927686*ro.*; 19. ·000353*da.*; 20. ·05783*cr.*; 21. 28·8469*lbs.*

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8. *Promiscuous Exercises in Decimals.*

1. Add together ·9854, 98·54, and 985·4; and subtract the sum from 9854. *Ans.* 8769·0746.

2. Multiply ·205 by ·205, and from the product subtract $\frac{3}{5}$ of $\frac{3}{3\frac{3}{5}}$. *Ans.* ·004525.

3. Divide ·025 by ·000625. *Ans.* 40.

4. Express ·00231 of a foot as the decimal of a mile. *Ans.* ·0000004375.

5. What decimal part of a £. is 16*s.* $11\frac{3}{4}d.$ $\frac{1}{5}$ *q.*? *Ans.* £ ·84916.

6. What decimal part of an acre is half a square yard? *Ans.* ·0001033 *ac.*

7. How many French metres, each equal to 39·371 English inches, are there in $30\frac{1}{4}$ English yards? *Ans.* 27·736.

8. How many imperial gallons, each 277·274 cubic inches, can be contained in a vessel whose capacity is $3\frac{1}{10}$ cubic feet? *Ans.* 19·3195 *gall.*

9. Calculate ·09365 cwt., at $10\frac{3}{4}d.$ per lb. *Ans.* 9*s.* $4\frac{3}{4}d.$+

10. Calculate £29. 15*s.* $6\frac{1}{2}d.$ at 16·09016*s.* per £. *Ans.* £23. 19*s.* 1·4*d.*

11. What decimal fraction of 36 da. 3 ho. 49 sec. is 3 da. 23 ho. 31 min.? *Ans.* ·1101674.

12. When the ounce of gold costs £3·89, what is the cost of ·04 lb.? *Ans.* £1. 17*s.* 4*d.*

13. Reckoning 5·07 yards to be worth 7·15 shillings, how many English ells are worth 5*s.* 6*d.*? *Ans.* 3·12.

14. How many bushels of corn, at ·617 dollar per bushel, would cost as much as 12½ barrels of flour, at 7·5 dollars per barrel? *Ans.* 148·906 *bu.*

15. A and B together manufacture 137·6¼ yards of cloth, B's quantity being 9·467 yards less than A's. How much is done by each? *Ans.* A, 73·5635 *yds.*

16. What number of works, each consisting of 2 volumes, and each volume 2·47 inches thick, would occupy 4 shelves, when 19 works, each 3 volumes, and each volume 1·3 inch thick, occupy 10 such shelves? *Ans.* 6.

17. What decimal of a lb. is equal to the difference between $\frac{5}{64}$ of a cwt. and ·004 of a ton? *Ans.* ·21 lbs.

18. Suppose that an engine of 8 horse power, working 10½ hours a-day, could raise from a certain depth a weight of 46·508 tons, in 6·795 days; what must be the horse power of an engine, working 5 days a week, for 8½ hours a-day, to raise 168·3 tons from the same depth in 5 weeks? *Ans.* 9·72 h. p.

19. The circumference of any circle divided by 3·14159 gives the diameter. Required a multiplier in lieu of that divisor. *Ans.* ·31831.

20. If 15·46 times A's money amount to 4·035 times B's, and ·144 of B's money is 23·19 crowns; how much money has each?

Ans. A £10. 10*s.* 1$\frac{7}{9}$*d.*; B £40. 5*s.* 2½*d.*

21. What number, having ·265 of its 21st part subtracted from it, will become equal to 11 times the difference of 2·9 and ·29? *Ans.* 29 $\frac{1}{13}$.

22. One agent can perform a work in ·91 hours; another agent can do the same in 1·05 hours. What portion of the work could be done by both these agents together in 15 minutes? Ans. $\frac{2}{3}\frac{3}{9}$.

23. Multiply 27·450867 by itself, employing the contracting process, so as to retain only four decimal figures in the product. *Ans.* 753·5501.

24. Multiply ·52368 by ·06975 retaining six decimal figures. *Ans.* ·036527.

25. Divide, by contraction, 4375 by 64964, retaining six decimal places. *Ans.* ·067345.

26. Divide, by contraction, ·01 cwt. by ·4657328 cwt., retaining five decimals. *Ans.* ·02147.

27. What vulgar fraction is the difference between ·$\dot{4}0\dot{7}$ and ·$\dot{4}0\dot{7}$? *Ans.* $\frac{1}{2\,4\,9\,7\,5}$.

28. Multiply 14·$\dot{0}3\dot{0}$ by ·$0\dot{1}3\dot{6}9\dot{8}6\dot{3}$. *Ans.* ·182.

29. Divide ·$\dot{3}6\dot{4}$ by 25·$\dot{0}0\dot{6}2\dot{3}$. *Ans.* ·014570942.

30. Compute 35·87 cwt. at 3·589*d* per lb. *Ans.* £60. 2*s.* $0\frac{4}{11}$*d.*

XVI. PRACTICE.

In calculating the cost of any considerable number of articles at a given price for each, it is usual to employ a compendious method, which, on account of its convenience and current use in business, has obtained the name of Practice.

Thus the cost of 136 yds. of cambric at 12*s.* 6*d.* the yd. is equal to 136 times 10*s.* + 136 times 2*s.* 6*d.*, *i. e.* 136 *halves* of £1 + 136 *eighths* of £1, or $\frac{1}{2}$ + $\frac{1}{8}$ of £136, or £68 + £17, = £85. The process has consisted in breaking up the given rate into parts which constitute primitive fractions of £1, then calculating for each part, and lastly combining the amounts.

Definition.—PRACTICE is the method of calculating amounts by means of aliquot parts. An aliquot part of a quantity is a part contained an exact number of times in that quantity.

In commencing instruction in this Rule such an example as the following may be selected:—

Examp. What cost 173 slates at 3*d.* each?

173*d.*	12)173*d.*	173*s.*
3		
12)519*d.*	14*s.* 5*d.*	3*d.* = ¼ = 43*s.* 3*d.* *Ans.*
	3	
43*s.* 3*d.*	43*s.* 3*d.*	

The teacher may show, first, that the whole amount must be 173 times 3*d.* = 3 times 173*d.*, and that it is expedient then to convert the amount of pence into shillings, &c.; secondly, that 173*d.*, may be converted into shillings and odd pence, and then taken 3 times; thirdly, that the method of Practice takes 173 times 3*d.* = ¼ of 173 times 1*s.* = ¼ of 173*s.*

The following Tables will serve for mental instruction and examination by the Teacher. When the table of the aliquots of £1 and 1*s.* is familiar to the Pupil, he may be conducted through the Exercises which we have subjoined to that Table: and it may be expedient that the Exercises in the Manual should be reserved for a promiscuous Praxis after he has worked all the classified sets which we shall here propose.

Table of Principal Aliquot Parts.

10*s.*	0*d.*	=	£½	2*s.*	6*d.*	=	£ 1/8	0*s.*	4*d.*	=	1/3*s.*
6	8	=	£⅓	2	0	=	£ 1/10	0	3	=	¼*s.*
5	0	=	£¼	1	8	=	£ 1/12	0	2	=	1/6*s.*
4	0	=	£⅕	1	0	=	£ 1/20	0	1½	=	1/8*s.*
3	4	=	£⅙	0	6	=	½ *s.*	0	1	=	1/12*s.*

Exercises.

1. 263 lbs of tea, at 5*s.* *Ans.* £65. 15*s.* 0*d.*
2. 365 days, at 1*s.* 8*d.* *Ans.* £30. 8*s.* 4*d.*
3. 1349 yds. of silk, at 2*s.* 6*d.* *Ans.* £168. 12*s.* 6*d.*
4. 153 weeks, at 10*s.* *Ans.* £76. 10*s.* 0*d.*
5. 221 copies of a book, at 6*s.* 8*d.* *Ans.* £73. 13*s.* 4*d.*
6. 1635 steam-boat fares, at 6*d.* *Ans.* £40. 17*s.* 6*d.*

7. 59 oranges, at 1½*d.* *Ans.* £0. 7*s.* 4½*d.*
8. 3657 tracts, at 1*d.* *Ans.* £15. 4*s.* 9*d.*
9. 157 lbs. of rice, at 4*d.* *Ans.* £2. 12*s.* 4*d.*
10. The particulars of a school account are, 47 reading-books at 3*s.* 4*d.*, 29 quires of paper at 1*s.*, 35 slates at 3*d.*, and 63 ditto at 2*d.* Required the amount.
Ans. £10. 4*s.* 11*d.*

When a given price is not an aliquot part of £1 or 1*s.*, it may be resolved into aggregate parts which are aliquots of those integers, and which may be used to calculate aggregate parts of the amount. But it often happens that one portion of a rate is a convenient aliquot of a previously used portion; and, accordingly, from the amount found by the one may be derived the amount for the other. Thus, the rate 11*s.* 3*d.* may be resolved into 10*s.* and 1*s.* 3*d.*, the former of which is £½, and the latter £$\frac{1}{16}$; but 16 is an inconvenient divisor; and, therefore, when we have found the amount at 10*s.* we should calculate for 1*s.* 3*d.* as the 8th part of 10*s.*, by taking the 8th part of the amount at 10*s.*

Examp. How much money will buy 263 pairs of shoes, at 9*s.* 7*d.* per pair?

The given rate is made up of 5*s.* + 3*s.* 4*d.* + 1*s.* 3*d.*, which are, respectively, £¼, £$\frac{1}{6}$, and ¼ of 5*s.*; or, it consists of 6*s.* 8*d.* + 2*s.* 6*d.* + 5*d.*, which are respectively, £$\frac{1}{3}$, £$\frac{1}{8}$, and $\frac{1}{6}$ of 2*s.* 6*d.* Hence, by setting out with the supposition that the price per pair is £1, which would make the amount £263, we may deduce the answer for the actual rate by either of the following forms:—

	£263				£263		
5*s.* 0*d.* = ¼	65	15	0	6*s.* 8*d.* = $\frac{1}{3}$	87	13	4
3 4 = $\frac{1}{6}$	43	16	8	2 6 = $\frac{1}{8}$	32	17	6
1 3 = $\frac{1}{16}$	16	8	9	0 5 = $\frac{1}{16}$	5	9	7
Ans.	£126	0	5		£126	0	5

Examp. Calculate 429 yards of calico at $7\frac{1}{2}d.$

Here, the given price may be resolved into $6d.$ and $1\frac{1}{2}d.$; or it may be derived as $\frac{1}{4}$ of the rate $2s.\ 6d.$

429*s.*		£429	
$6d. = \frac{1}{2}$	214*s.* 6*d.*	$2s.\ 6d. = \frac{1}{8}$	53 12 6
$1\frac{1}{2}d. = \frac{1}{4}$	53 $7\frac{1}{2}$	$0\ 7\frac{1}{2} = \frac{1}{4}$	£13. 8*s.* $1\frac{1}{2}d.$
20)268 $1\frac{1}{2}$			
Ans. £13. 8*s.* $1\frac{1}{2}d.$			

Let the pupil be made familiar with the following promiscuous aliquots, and then proceed to the Exercises that are subjoined.

Table of Subordinate Aliquot Parts.

2*s.* 6*d.*	= $\frac{1}{4}$ of 10*s.* 0*d.*												
1 8	= $\frac{1}{8}$ of 10 0												
1 4	= $\frac{1}{8}$ of 4 0												
1 3	= $\frac{1}{4}$ of 5 0	= $\frac{1}{8}$ of 10*s.* 0*d.*											
0 10	= $\frac{1}{4}$ of 3 4	= $\frac{1}{6}$ of 5	0	= $\frac{1}{12}$ of 10*s.* 0*d.*									
0 8	= $\frac{1}{3}$ of 2 0	= $\frac{1}{6}$ of 4	0	= $\frac{1}{10}$ of 6	8								
0 $7\frac{1}{2}$	= $\frac{1}{2}$ of 1 3	= $\frac{1}{4}$ of 2	6	= $\frac{1}{8}$ of 5	0								
0 6	= $\frac{1}{2}$ of 2 6	= $\frac{1}{3}$ of 4	0	= $\frac{1}{10}$ of 5	0								
0 $3\frac{3}{4}$	= $\frac{1}{2}$ of 0 $7\frac{1}{2}$	= $\frac{1}{4}$ of 1	3	= $\frac{1}{8}$ of 2	6								
0 $2\frac{1}{2}$	= $\frac{1}{6}$ of 1 3	= $\frac{1}{8}$ of 1	8	= $\frac{1}{12}$ of 2	6								
0 $1\frac{1}{2}$	= $\frac{1}{4}$ of 0 6	= $\frac{1}{10}$ of 1	3	= $\frac{1}{20}$ of 2	6								
0 $1\frac{1}{4}$	= $\frac{1}{8}$ of 0 $7\frac{1}{2}$	= $\frac{1}{6}$ of 0	10	= $\frac{1}{12}$ of 1	3								
0 $0\frac{3}{4}$	= $\frac{1}{3}$ of 0 $1\frac{1}{2}$	= $\frac{1}{4}$ of 0	3	= $\frac{1}{8}$ of 0	6								
ditto	= $\frac{1}{10}$ of 0 $7\frac{1}{2}$	= $\frac{1}{20}$ of 1	3	= $\frac{1}{40}$ of 2	6								

Exercises.

1. Calculate 4049, at $8\frac{1}{2}d.$; at $7\frac{1}{4}d.$; at $6\frac{3}{4}d.$

Ans. £143. 8*s.* $0\frac{1}{2}d.$; £122. 6*s.* $3\frac{1}{2}d.$; £113. 17*s.* $6\frac{3}{4}d.$

2. Calculate 197, at 2*s.* 11*d.*; at 1*s.* $10\frac{1}{2}d.$; at 15*s.* 6*d.*

Ans. £28. 14*s.* 7*d.*; £18. 9*s.* $4\frac{1}{2}$d.; £152. 13*s.* 6*d.*

3. Calculate 2319, at 5*d.*; at 7*d.*; at $\frac{1}{4}d.$
Ans. £48. 6*s.* 3*d.*; £67. 12*s.* 9*d.*; £7. 4*s.* $11\frac{1}{4}d.$

4. Calculate 2000, at 2*s.* $9\frac{1}{4}d.$; at 4*s.* 2*d.*; at 10*s.* 8*d.*
Ans. £281. 5*s.*; £416. 13*s.* 4*d.*; £1066. 13*s.* 4*d.*

5. Calculate 146, at 14*s.* 8*d.*; at 15*s.* $2\frac{1}{4}d.$; at 7*s.* $9\frac{1}{2}d.$
Ans. £107. 1*s.* 4*d.*; £110. 17*s.* $4\frac{1}{2}d.$; £56. 17*s.* 7*d.*

6. Calculate 365, at 3*s.* $9\frac{1}{2}d.$; at 16*s.* $7\frac{3}{4}d.$; at 13*s.* $2\frac{1}{2}d.$
Ans. £69. 3*s.* $11\frac{1}{2}d.$; £303. 15*s.* $8\frac{3}{4}d.$; £241. 1*s.* $0\frac{1}{2}d.$

When the given price falls short of £1, or 1*s.*, &c., by some aliquot part of these integers, it is often expedient to calculate for that aliquot, and subtract; as in the following examples, in which we have deduced the answers from the amounts at £1. and 10*s.*

Examp. 257 pairs of boots at 17*s.* 6*d.* and at 9*s.* 2*d.*

	£.	s.	d.
2*s.* 6*d.* = $\frac{1}{8}$)257	0	0	
Amt. at 2*s.* 6*d.* = 32	2	6	
Ans. £224.	17	6	

	£.	s.	d.
10*d.* = $\frac{1}{17\frac{1}{2}}$)128	10	0	
Amt. at 10*d.* = 10	14	2	
Ans. £117	15	10	

The following Table may assist in detecting opportunities of such abbreviation:—

Table of Aliquot Complements.

19*s.*	6*d.*	wants $\frac{1}{40}$	of £1.		
19	4	.... $\frac{1}{80}$	.....		
19	0	.... $\frac{1}{20}$	.....		
18	4	.... $\frac{1}{12}$	.....		
18	0	.... $\frac{1}{10}$	.....		
17	6	.... $\frac{1}{8}$	.....		
16	8	.... $\frac{1}{6}$	.....		
16	0	.... $\frac{1}{5}$	.....		

15*s.*	0*d.*	wants $\frac{1}{4}$	of £1.		
13	4	.... $\frac{1}{8}$	.....		
	11	.... $\frac{1}{17\frac{1}{2}}$	of 1*s.*		
	$10\frac{1}{2}$	.... $\frac{1}{8}$	....		
	10	.... $\frac{1}{8}$	....		
	9	.... $\frac{1}{4}$	....		
	8	.... $\frac{1}{3}$	....		

Exercises.

1. Calculate 470, at 15*s.*; at 10½*d.*; at 16*s.* 8*d.*
Ans. £352. 10*s.*; £20. 11*s.* 3*d.*; £391. 13*s.* 4*d.*

2. Calculate 532, at 18*s.* 4*d.*; at 10*d.*; at 8*s.* 9*d.*
Ans. £487. 13*s.* 4*d.*; £22. 3*s.* 4*d.*; £232. 15*s.*

3. Calculate 2345, at 19*s.* 8*d.*; at 18*s.* 6*d.*; at 6*s.* 7*d.*
Ans. £2305. 18*s.* 4*d.*; £2169. 2*s.* 6*d.*; £771. 17*s.* 11*d.*

When the given price is an exact number of shillings, it is generally expedient to express it in 10ths of £1, and multiply by it. Thus, 47 yards at 16*s.* = 47 times £·8 = £37·6, or £37. 12*s.* Thus, also, 59 yards at 17*s.* = 59 times £·8½ = £50·1½, or £50. 3*s.*

The Pupil may now proceed to work the Exercises in the Manual, as the illustrations we have yet to give will not become requisite till he has arrived at Ex. 18.

When the given price contains pounds, multiply by the number of pounds, and to the product add the amounts for the smaller denominations, if any, calculated as aliquot parts.

Examp. Find the worth of 573 oz. of gold at £3. 17*s.* 9*d.*; and of 267 cwt. of sugar at £2. 14*s.* 6*d.*

	£	*s.*	*d.*		£	*s.*	*d.*
	573				267		
	3				2		
	1719	0	0		534		
15*s.* 0*d.* = ¼	429	15	0	14*s.* 0*d.* = £·7	186	18	0
2 6 = ⅙	71	12	6	0 6 = $\frac{1}{40}$	6	13	6
0 3 = $\frac{1}{120}$	7	3	3				
Ans.	2227	10	9	*Ans.*	727	11	6

When the quantity whose value is required contains a fraction, or several denominations, convert it into £. *s.* *d.* as the amount at £1, and then proceed in the usual way. If the fraction or inferior denominations be

not convertible into proper money, neglect them till the main integral quantity is computed, and then apply them in aliquot parts to the given rate.

Examp. Calculate 76⅝ ells at 13*s.* 6*d.*; and 53 ac. 3 ro. 15 po. at £2. 10*s.* 6*d.* per acre.

	£76.	7*s.*	6*d.*
10*s.* 0*d.* = ½	38	3	9
3 4 = ⅓	12	14	7
0 2 = 1/20	0	12	8¾
Ans.	£51	11	0¾

	£53.	16*s.*	10½*d.*
			2
	107	13	9
10*s.* 0*d.* = ½	26	18	5½
0 6 = 1/40	1	6	11 1/16
Ans.	£135	19	1 5/16

Examp. Calculate 373¼ at £12. 15*s.* 9*d.*; and 6 cwt. 1 qr. 17 lb. at 56*s.* 10½*d.* per cwt.

	£373		
	12		
	4476		
12*s.* 0*d.* = 1/20	223	16	0
3 0 = ¼	55	19	0
0 9 = ¼	13	19	9
¾ = ⅓ of rate	4	5	3
⅓ of do.	1	8	5
Ans.	£4775	8	5

	£2.	16*s.*	10½*d.*
			6
	17	1	3
1 qr. = ¼	0	14	2½ ½
16 lb. = 1/7	0	8	1½
1 lb. = 1/28	0	0	6 0/4 ⅜
Ans.	£18	4	1 0/4 ⅞

When absolute accuracy is not required, the compound quantity, though not entirely convertible into proper money, may be changed into an approximate amount for £1. by a decimal extension of the pence to two or three figures. In converting cwt. qrs. lbs. into their value at £1. per cwt., the qrs. are valued at 5*s.*, and the lbs. at 2¼*d.* See the first of the subjoined examples.

The Rule of Practice is applicable to other calculations besides money rates. See the second of the *subjoined examples*:

Examp. Required the amount of 91 cwt. 3 qrs. 13 lbs. of hops, at £9. 13*s.* 7*d.* per cwt.

Examp. What quantity of grain is reaped from 736 ac. 3 ro. 28 po., the average produce being 4 qrs. 5 bu. 1 pk. per acre?

£91. 17*s.* 3·857*d.*
9

	826	15	10·71
10*s.* 0*d.* = ½	45	18	7·93
3 4 = ⅓	15	6	2·64
0 3 = 1/40	1	2	11·60
Ans. £889	3		8·88 or 9*d.*

736 qrs. 7 bu. 1·6 pks.
4

	2947	5	2·4
4 bu. = ¼	368	3	2·8
1 bu. = ¼	92	0	3·7
1 pk. = ¼	23	0	0·9
Ans. Qrs.3431	2		1.8 or 2 pks.

A ready perception of the expedients best adapted to particular quantities and prices can only be attained by practised reflexion.

Exercises.

1. 467 at 1*d.*; 342 at 1½*d.*; 275 at 2*d.*
   Ans. £1. 18*s.* 11*d.*; £2. 2*s.* 9*d.*; £2. 5*s.* 10*d.*
2. 376 at 3*d.*; 491 at 4*d.*; 276 at 6*d.*
   Ans. £4. 14*s.*; £8. 3*s.* 8*d.*; £6. 18*s.*
3. 95 at 5*d.*; 69 at 7*d.*; 57 at 8*d.*
   Ans. £1. 19*s.* 7*d.*; £2. 0*s.* 3*d.*; £1. 18*s.*
4. 83 at 9*d.*; 78 at 10*d.*; 85 at 11*d.*
   Ans. £3. 2*s.* 3*d.*; £3. 5*s.*; £3. 17*s.* 11*d.*

5. 2375 at 2½*d.*; 3649 at 4½*d.*; 2857 at 7½*d.*
Ans. £24. 14*s.* 9½*d.*; £68. 8*s.* 4½*d.*; £89. 5*s.* 7½*d.*

6. 365 at 10½*d.*; 577 at 2¼*d.*; 3065 at 3¾*d.*
Ans. £15. 19*s.* 4½*d.*; £5. 8*s.* 2¼*d.*; £47. 17*s.* 9¾*d.*

7. 247 at 5½*d.*; 1294 at 10½*d.*; 469 at 9½*d.*
Ans. £5. 13*s.* 2½*d.*; £55. 5*s.* 3½*d.*; £18. 11*s.* 3½*d.*

8. 259 at 1*s.*; 376 at 2*s.*; 423 at 4*s.*
Ans. £12. 19*s.*; £37. 12*s.*; £84. 12*s.*

9. 739 at 6*s.*; 675 at 8*s.*; 732 at 10*s.*
Ans. £221. 14*s.*; £270.; £366.

10. 67 at 5*s.*; 79 at 15*s.*; 326 at 18*s.*
Ans. £16. 15*s.*; £59. 5*s.*; £293. 8*s.*

11. 755 at 6*s.* 8*d.*; 291 at 3*s.* 4*d.*; 427 at 2*s.* 6*d.*
Ans. £251. 13*s.* 4*d.*; £48. 10*s.*; £53. 7*s.* 6*d.*

12. 673 at 1*s.* 8*d.*; 379 at 8*s.* 4*d.*; 365 at 5*s.* 10*d.*
Ans. £56. 1*s.* 8*d.*; £157. 18*s.* 4*d.*; £106. 9*s.* 2*d.*

13. 196 at 7*s.* 6*d.*; 223 at 9*s.* 8*d.*; 3706 at 17*s.* 6*d.*
Ans. £73. 10*s.*; £107. 15*s.* 8*d.*; £3242. 15*s.*

14. 2354 at 16*s.* 8*d.*; 3290 at 18*s.* 6*d.*; 553 at 19*s.* 6*d.*
Ans. £1961. 13*s.* 4*d.*; £3043. 5*s.*; £539. 3*s.* 6*d.*

15. 468 at 2*s.* 10½*d.*; 393 at 1*s.* 8¼*d.*; 67 at 3*s.* 6½*d.*
Ans. £67. 5*s.* 6*d.*; £33. 3*s.* 2¼*d.*; £11. 17*s.* 3½*d.*

16. 209 at 4*s.* 8½*d.*; 87 at 5*s.* 6¾*d.*; 129 at 8*s.* 0½*d.*
Ans. £49. 4*s.* 0½*d.*; £24. 3*s.* 11¼*d.*; £51. 17*s.* 4½*d.*

17. 569 at 9*s.* 11¼*d.*; 467 at 13*s.* 5½*d.*; 753 at 15*s.* 4¾*d.*
Ans. £282. 14*s.* 5¼*d.*; £314. 5*s.* 0½*d.*; £579. 13*s.* 0¾*d.*

18. 1087 at £2. 18*s.*; 390 at £5. 9*s.* 4½*d.*; 186 at £12. 12*s.* 7*d.*
Ans. £3152. 6*s.*; £2132. 16*s.* 3*d.*; £2349. 0*s.* 6*d.*

19. 27¾ at 4*s.* 9*d.*; 46⅓ at 27*s.* 10*d.*; 37⅝ at 5½*d.*
Ans. £6. 11*s.* 9¾*d.*; £64. 14*s.* 3*d.*; 17*s.* 2¾*d.* ¾.

20. 4 cwt. 3 qrs. 14 lbs. at £2. 14*s.* 6*d.* per cwt.
Ans. £13. 5*s.* 8¼*d.*

21. 26 cwt. 1 qr. 10 lbs. at £3. 11*s.* 2*d.* per cwt.
Ans. £93. 14*s.* 5$\frac{3}{4}$*d.*

22. 18 cwt. 2 qrs. 25 lbs. at £0. 18*s.* 7$\frac{1}{2}$*d.* per cwt.
Ans. £17. 8*s.* 8$\frac{1}{2}$*d.*$\frac{1}{5}\frac{1}{6}$.

23. 26 ac. 3 ro. 15 po. at £1. 12*s.* 6*d.* per acre.
Ans. £43. 12*s.* 5$\frac{1}{1}\frac{1}{6}$*d.*

24. £84. 17*s.* 9*d.* at 4*s.* 6$\frac{1}{2}$*d* per £.
Ans. £19. 5*s.* 6$\frac{1}{4}$*d.* $\frac{1}{4}\frac{9}{6}q$.

25. 53 yds. 2 qu. 3 na. at 5*s.* 10$\frac{1}{2}$*d.* per yard.
Ans. £15. 14*s.* 3$\frac{1}{2}$*d.* $\frac{3}{1}\frac{}{6}q$.

26. 29 ho. 31 min. at 4 mi. 57 yds. per hour.
Ans. 119*mi.* 39*yds.* 2*ft.* 4$\frac{1}{5}$*in.*

27. 123·541 cubic feet, at 6*s.* 10*d.* per foot.
Ans. £42. 4*s.* 2·362*d.*

Bill of Parcels.

28. London, Aug. 1st, 1851. Messrs. Willis and Mason bought of Mr. Charles Ward—274 yds. Scotch linen, at 3*s.* 5$\frac{1}{2}$*d.*; 239 yds. do., at 2*s.* 11*d.*; 197 yds. Irish linen, at 2*s.* 3$\frac{1}{2}$*d.*; and 175 yds. do., at 1*s.* 9$\frac{3}{4}$*d.* Make out the bill, computing by two methods;—1st, by the usual method of aliquot parts; 2dly, by multiplying by each price as a vulgar fraction of a £., and extending each amount in decimals of a £.
Ans. £120. 13*s.* 3$\frac{3}{4}$*d.*

XVII. PER CENTAGES.

In many of the calculations of business and science, the number 100 is employed, as a constant denominator in the expression of rates or ratios. This is to promote facility of computation and comparison.

Thus, if an agent transact business for his employer to the amount of £1575, the remuneration for such

agency will be *easily calculated*, if it be fixed at an allowance of so much for every £100. in the amount, say £2. for every £100, or 2 *per cent.* as it is called; then, the agent will receive $\frac{2}{100}$ of £1575 = £15·75 × 2 = £31·5, or £31. 10*s.*

Again, If A invests in trade £250, and gains £7. 10*s.*, while B invests £40, and gains £1. 8*s.*; then A's rate of gain is $\frac{7\frac{1}{2}}{250}$ of his outlay = $\frac{3}{100}$, or 3 per cent.; B's rate of gain is $\frac{1\frac{2}{3}}{40}$ of his outlay = $\frac{7}{200}$ = $\frac{3\frac{1}{2}}{100}$ or 3½ per cent. So that in the per centage rates, 3 and 3½, we have a *direct comparison* between the gains of A and B.

The calculation of per centages may be performed either by first principles, or by Proportion statement.

Examp. If my gain by selling cloth be £7½ on every £100. which I lay out upon it; what sum do I clear by the sale of a quantity which cost me £260. 15*s.*

By First Principles.

£7½ gained by £100; therefore, $\frac{£7\frac{1}{2}}{100}$ gained by £1.;

and therefore $\frac{£7\frac{1}{2} \times 260{\cdot}75}{100}$ or $\frac{£260.\ 15s. \times 7\frac{1}{2}}{100}$ gained by £260. 15*s.*

That is, $\frac{3}{40}$ of £260. 15*s.* = £19. 11*s.* 1½*d.* *Ans.*

By Proportion Statement.

£100 : £260. 15*s.* :: £7½ : £19. 11*s.* 1½*d.* *Ans.*

The above question would be more concisely expressed thus:—Required the gain on £260. 15*s.* at 7½ per cent.

And it should be observed, that though the rate thus expressed is generally understood to mean £7½ on £100, *it may also signify* 7½*s.* on 100*s*, or 7½*d.* on 100*d.*, and

denotes generally $7\frac{1}{2}$ *hundredths* of the given sum. However, if the assigned rate is in shillings per cent., as 7*s*. 6*d*. per cent., the conventional understanding is 7*s*. 6*d*. per £100, which is £$\frac{3}{8}$ on £100, or generally, $\frac{3}{8}$ per cent.

It is often advisable to retain the denominator 100, rather than reduce the fraction to lower terms.

Examp. Required the amount of 6 per cent. on £326. 14*s*. 8½*d*.

Here $\frac{6}{100} = \frac{3}{50}$; but the original fraction is the more convenient. The solution, accordingly, will be performed as in the margin, by multiplying by 6 and dividing by 100, the division being very simply performed by cutting off two right hand figures as the remainder, and taking the figures on the left of it as the quotient.

£326. 14*s*. 8½*d*.
6

£19,60 8 3
20

s.12,08
12

d.,99

Ans. £19. 12*s*. 1*d*.

When a rate is given in *guineas* per cent., £100. is the understood relative integer. Thus, for $3\frac{1}{2}$ guineas per cent. the fractional multiplier would be $\frac{3\frac{1}{2} \times 21}{100 \times 20} = \frac{147}{4000}$. The calculation, however, may proceed as if the guineas were pounds, and then the required correction will be made by adding a 20th part. Or we may first increase the given sum by its 20th part, and then calculate with the given rate as for £.

The preceding explanations will prepare for the working of the first 16 Exercises, which are followed by questions of a less elementary character. For the latter kind, the following preparatory exemplification may be requisite :—

Examp. Bought tea at 4*s*. 8*d*. a lb. What is my gain p. c. by selling it at 5*s*. 3*d*.?

That is, if 56*d*. gains 7*d*., what does £100. gain?

56*d*. : 7*d*. :: £100. : £12½ p. c. Ans.

Examp. At what rate per cent. of shrinkage will 97 cwt. 3 qrs. 15 lbs. of tobacco lose 3 cwt. $2\frac{3}{4}$ lbs. by drying?

That is, If 10963 lbs. lose 338·75 lbs., what does 100 lose?

10963 lbs. : 338·75 lbs. :: 100 : 3·09 p. c. *Ans.*

Examp. What quantity augmented by $3\frac{1}{2}$ p. c. becomes 186?

$\frac{3\frac{1}{2}}{100} = \frac{1}{28}$ of the quantity; $1\frac{1}{28}$ of the quantity = 186.

$1\frac{1}{28} : 1 :: 186 : 30 \times 6 = 180$. *Ans.*

Otherwise:—If 100 becomes $103\frac{1}{2}$, what quantity, at the same rate of increase, becomes 186?

$103\frac{1}{2} : 100 :: 186 : 180$. *Ans.*

Examp. An account is rendered, for present payment of which $4\frac{1}{2}$ p. c. discount is allowed, making the net cash amount £15. 10*s.* $4\frac{1}{2}$*d.* Required the amount rendered.

The allowance is $4\frac{1}{2}$ hundredths of the amount rendered; therefore the cash payment is the remaining $95\frac{1}{2}$ hundredths of the amount rendered, or is to the amount rendered as $95\frac{1}{2}$ to 100; whence,

$95\frac{1}{2} : 100 ::$ £15. 10*s.* $4\frac{1}{2}$*d.* : £16. 5*s.* *Ans.*

Examp. If I gain 6 p. c. by selling goods at 12*s.* 6*d.*, what is my gain or loss p. c. by selling them at 9*s.* 6*d.*?

That is, if £100 is made £106 by selling at 12*s.* 6*d.*, what is £100 made by selling at 9*s.* 6*d.*?

12*s.* 6*d.* : 9*s.* 6*d.* :: 106 : 80·56

100 − 80·56 = 19·44 p. c. loss. *Ans.*

Otherwise:—£106 ÷ 12*s.* 6*d.* is the number of yards I bought for £100; therefore this quantity calculated at 9*s.* 6*d.* must be the amount I shall otherwise receive for what cost £100; that is £106 ÷ $12\frac{1}{2} \times 9\frac{1}{2}$ = £80·56; &c.

Exercises.

1. An agent purchases goods, on account of his employer, to the amount of £1356. 12*s.* 6*d.* Required his commission, at 3 per cent. *Ans* £40. 13*s.* 11·7*d.*

2. What is the commission on £435. at $4\frac{1}{2}$ per cent.? *Ans.* £19. 11*s.* 6*d.*

3. What is the brokerage on £570. at 7*s.* 6*d.* per cent.? *Ans.* £2. 2*s.* 9*d.*

4. Calculate $\frac{1}{4}$ per cent. commission on £256. 10*s.* 6*d.* *Ans.* £1. 5*s.* $7\frac{3}{4}d.\frac{8}{25}$.

5. Required the premium of insurance on £750. at $2\frac{1}{2}$ per cent. *Ans.* £18. 15*s.*

6. What sum will insure £860. from Leith to London at 15*s.* 6*d.* per cent.? *Ans.* £6. 13*s.* $3\frac{3}{4}d.$

7. A vessel and freight are insured to the amount of £1500. How much is the premium, at $2\frac{1}{4}$ guineas per cent.? *Ans.* £35. 8*s.* 9*d.*

8. Allow a discount of 5 per cent. off £13. 5*s.* 8*d.*: What remains? *Ans.* £12. 12*s.* $4\frac{4}{5}d.$

9. What cash will discharge an account rendered at £12. 10*s.* $6\frac{1}{4}d.$, if 4 per cent. discount be allowed for present payment? *Ans.* £12. 0*s.* $6\frac{6}{25}d.$

10. Required the amount of 56 per cent. of pure silver, in 436 lbs. 10 oz. of silver ore.
Ans. 244*lbs.* 7*oz.* 10*dwt.* $9\frac{3}{5}grs.$

11. If 360 qrs. of corn lose, by drying, $7\frac{1}{2}$ per cent. in measure; what quantity remains? *Ans.* 332·64*qrs.*

12. I cleared 5 per cent. on an outlay of £22. 17*s.* 6*d.* Required the amount of the return. *Ans.* £24. 0*s.* $4\frac{1}{2}d.$

13. Bought tea at 4*s.* 8*d.* per lb. How must I sell it to gain $12\frac{1}{2}$ per cent.? *Ans.* 5*s.* 3*d.*

14. I gave £30. for 85 lbs. of tea. How must I retail it per lb. to gain $8\frac{1}{3}$ per cent.? *Ans.* 7*s.* $7\frac{1}{17}d.$

15. I bought cloth at 10*s.* 6*d.*, and sold it at a loss of $28\frac{4}{7}$ per cent. Required the selling price. *Ans.* 7*s.* 6*d.*

16. What is the gain on 5*s.* $1\frac{1}{2}d.$ at 20 per cent.?
Ans. 1*s.* $0\frac{1}{2}d.$

17. The former population of a parish was 3760; its present population is 3925. Required the increase per cent. *Ans.* 4·3883 *p.c.*

18. Bought cloth at 6*s.* 3*d.*, and sold it at 7*s.* 5*d.*: what was my gain per cent.? *Ans.* 18$\frac{4}{7}$ *p.c.*

19. Bought 40 yards of cloth at 7*s.* 5*d.* the yard, and sold them at 6*s.* 3*d.*: what was my loss per cent.? *Ans.* £15. 14*s.* 7$\frac{2}{3}\frac{1}{3}$*d.*

20. At what rate per cent. does the commission on £780. amount to £6. 16*s.* 6*d.*? *Ans.* $\frac{7}{8}$ or 17*s.* 6*d.*

21. Insured £3500. by paying £128. 12*s.* 6*d.* premium: required the rate per cent. in guin. *Ans.* 3$\frac{1}{2}$*g.*

22. If by selling coffee at 2*s.* 8*d.* per lb. I gain 20 p. c.; what has been the prime cost? *Ans.* 2*s.* 2$\frac{2}{3}$*d.*

23. What sum, reduced by a discount of 8 p. c., becomes 9*s.* 7*d.*? *Ans.* 10*s.* 5*d.*

24. How must I rate sugar per lb., to get 9$\frac{3}{4}$*d.* after allowing 2$\frac{1}{2}$ p. c. discount? *Ans.* 10*d.*

25. Gained 15 p. c. by selling 500 yds. of linen for £138. What did a yard cost me? *Ans.* 4*s.* 9$\frac{1}{2}$*d.*

26. Gained 4 p. c. by selling goods at 13*s.* 4*d.* How much p. c. should I have gained by selling at 15*s.*? *Ans.* 17 *p.c.*

27. Lost 1 per cent. by selling coffee at 2*s.* 3*d.* What did I gain or lose p. c. when it rose to 2*s.* 6*d.*? *Ans.* 10 *p.c. gain.*

28. A merchant sells tea at 7*s.* 8*d.*, and gains 15 p. c. How much will he gain, if the price rise to 8*s.* 4*d.*? and how much will he gain or lose, if it fall to 6*s.* 8$\frac{1}{2}$*d.*? *Ans.* 25 *p.c. gain*; $\frac{1}{5}$ *p.c. gain.*

29. At what must I profess to sell broadcloth, which cost me 19*s.* a yard, that, if I should be obliged to allow 5 per cent. off, my profit may be 25 p. c.? *Ans.* 25*s.*

XVIII. PER CENTAGES ON TIME.

Having, in the preceding article, explained the principles of the calculation of *absolute* per centages, we can easily extend the application of those principles to questions in which the rate per cent. is *affected by the consideration of Time.*

If £52. 10*s.*, due to me at present, remains unpaid for a year, I may charge 5 per cent. for the year's loan of the money, and shall thus receive £2. 12*s.* 6*d.* of remuneration, making the amount £55. 2*s.* 6*d.* If the money is unpaid for two, three, &c. years, I shall charge 10 p. c., 15 p. c., &c. on £52. 10*s.* Uniformly, I shall charge 5 p. c. multiplied by the number of years.

Per Centages on Time, therefore, are at once convertible into absolute per centages. 4 p. c. yearly, for 3 years, is 12 p. c. altogether; 3½ p. c. yearly, for ½ a year is just 1¾ p. c.

Per Centages on Time belong chiefly to two departments of Arithmetic, called *Interest* and *Discount.*

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1. SIMPLE INTEREST.

Definitions.—INTEREST is payment received for the loan of money: and is calculated at a certain rate per cent. per annum. *Per annum* signifies *for a year.*

The money that yields interest is called the Principal, and, when combined with its interest, is called the Amount. When interest is charged on one Principal it is called *Simple* Interest.

If interest, when it becomes due, is added to the Principal, and allowed to bear interest along with it, there arises what is called *Compound* Interest.

When a debt remains unpaid beyond the stipulated term of credit, the creditor cannot, in this country, claim legally a higher

rate of interest than 5 per cent. per annum. In former periods of our history the legal rate has been much higher, and it has generally been reduced in proportion as commerce has extended. In Ireland, the United States of America, and some other places, 6 per cent. is the legal interest.

The *market rate* of interest in Britain has been for many years lower than the legal rate.

Examp. Required the interest on £15. 12*s.* 6*d.* for a year, at 5 p. c. per annum.

5 p. c. = $\frac{5}{100}$ = $\frac{1}{20}$ of the principal = 15*s.* 7½*d.* *Ans.*

Examp. Find the interest on £165. 15*s.* for 4 years, at 5 p. c. per ann.

5 p. c. per ann. × 4 yrs. = 20 p. c. = $\frac{20}{100}$ = $\frac{1}{5}$.

£165. 15*s.* × $\frac{1}{5}$ = £33. 3*s.* *Ans.*

Examp. What is the amount of £426, for 3¾ years, at 3½ p. c. per ann.?

3½ × 3¾ = 13⅛ p. c., or 13⅛ *hundredths;*

4·26 × 13⅛ = 4·26 × 105 ÷ 8 = £55. 18*s.* 3*d.* Int.

£426 + Int. = £481. 18*s.* 3*d.* Amount. *Ans.*

Or, the amount might be found thus: The amount of £100 is £113⅛; therefore,

100 : 113⅛ :: £426 : *Ans.*

Although it is expedient that the pupil be well accustomed to regard the product of the rate by the time (in years) as the absolute per cent., it may be pointed out to him, that in many instances the rate and time should be used as two multipliers rather than in composition. Thus, if the rate is 4¼ and the time 7 yrs., it is better to multiply by 7 and by 4¼ as factors than by 29¾, because the multiplier 29¾ hundredths does not easily admit of simplification. The best general method is to place the rate and the number of years, as factors over the denominator 100, and then to cancel so far as may be possible and convenient.

Months must be reckoned as 12ths of a year, and *Days* as 365ths, because the rate is always *per annum.*

In computing the number of days intervening from one date to another, we reckon the commencing and the concluding day as one, unless it be otherwise specified. Thus, from 12th March to 21st July, we have 19 + *30* + *31* + 30 + 21 = 131 days.

Examp. What is the amount of £257. 10*s.* 6*d.*, lent on 16th June, 1851, and repaid on 2nd March, 1852, with interest at 5 p. c.?

From 16th June to 2nd March, leap year, = 260 days; 5 p. c. for $\frac{260}{365}$ of a year = $\frac{260}{365}$ of $\frac{1}{20}$ of the Principal; $\frac{1}{73}$ of £257. 10*s.* 6*d.* = £9. 3*s.* $5\frac{11}{365}$*d.* Int.

257. 10 6 Princ.

Ans. £266. 13 $11\frac{11}{365}$ Amt.

When the Principal, the Rate, or the Time, is a required quantity, as in the Exercises following the 24th, employ a Proportion statement, as in the following examples:—

Examp. What principal will gain £17. 17*s.* in 4 years, at $3\frac{1}{2}$ p. c.?

The interest for one year is $\frac{1}{4}$ of £17·85; therefore, If £100 gain £$3\frac{1}{2}$ in a year, what principal will gain $\frac{1}{4}$ of £17·85 in the same time?

$3\frac{1}{2}$: $\frac{1}{4}$ of 17·85 :: £100. : $\frac{1}{2}$ of 1785 ÷ 7 = £127·5. *Ans.*

Examp. At what rate p. c. per ann. will £127. 10*s.* gain £17. 17*s.* in 4 years?

Here, the question is, If £127. 10*s.* gain $\frac{1}{4}$ of £17·85 in a year, what will £100 gain in the same time?

127·5 : 100 :: $\frac{1}{4}$ of 17·85 : 1785 ÷ 510 = $3\frac{1}{2}$ p. c. *Ans.*

Examp. In what time will £127. 10*s.* gain £17. 17*s.* at $3\frac{1}{2}$ p. c. per ann.?

That is, If £100. gain £3·5 in one year, in what time will £127·5 gain £17·85?

$$\left.\begin{matrix} 127\cdot5 : 100 \\ 3\cdot5 : 17\cdot85 \end{matrix}\right\} :: 1 \text{ yr.} : 4 \text{ yrs. } \textit{Ans.}$$

Examp. What principal will amount to £145. 7*s.* in 4 years, at $3\frac{1}{2}$ p. c. per ann.?

The absolute gain on £100 is £14; therefore, if

£100 amount to £114, what principal will amount to £145. 7*s.*?

114 : 145·35 :: £100 : £127. 10*s.* *Ans.*

EXERCISES.

	£.	*s.*	*d.*		Yrs.		P.C.	*Answers.* £.	*s.*	*d.*
1. Int. on	540	0	0	for	1	at	4	21	12	0
2.	387	10	0	..	1	..	5	19	7	6
3.	2750	0	0	..	3	..	5	412	10	0
4.	1267	12	6	..	5	..	4	253	10	6
5.	762	17	0	..	4	..	$3\frac{1}{2}$	106	15	$11\frac{3}{4}\ \frac{1}{8\,5}$
6.	153	6	8	..	5	..	5	38	6	8
7.	32	13	4	..	7	..	$4\frac{1}{4}$	9	14	$4\frac{1}{4}\ \frac{3}{5}$
8.	165	6	$2\frac{1}{2}$	..	6	..	5	49	11	$10\frac{1}{4}\ \frac{2}{5}$
9.	1726	5	$3\frac{1}{2}$	..	$7\frac{1}{2}$	..	4	517	17	$7\frac{1}{2}\ \frac{1}{8\,0}$
10.	440	18	7	..	$3\frac{1}{2}$	..	$2\frac{1}{2}$	38	11	$7\frac{1}{2}\ \frac{1}{8\,0}$
11.	59	15	7	..	$5\frac{1}{3}$	..	$7\frac{1}{2}$	23	18	$2\frac{3}{4}\ \frac{1}{5}$
12.	23	17	8	..	$\frac{1}{4}$	..	$6\frac{1}{4}$	0	7	$5\frac{1}{2}\ \frac{1}{4}$
13.	1	19	$10\frac{1}{2}$	..	$\frac{3}{4}$	..	$3\frac{1}{2}$	0	1	$0\frac{8\,9\,7}{1\,6\,0\,0}$
14.	325	5	6	..	$2\frac{1}{8}$	..	10	69	2	$5\frac{1}{4\,0}$

15. What is the amount of £563. 12*s.* for $6\frac{3}{4}$ yrs. at £$3\frac{3}{4}$ per cent. per annum? *Ans.* £706. 5*s.* $2\frac{7}{10}$*d.*

16. Required the interest of £1380. for 5 months, at 5 p. c. per annum. *Ans.* £28. 15*s.*

17. Required the amount of £236. 17*s.* 6*d.* for 7 months, at $4\frac{1}{2}$ p. c. per annum. *Ans.* £243. 1*s.* $10\frac{1}{2}$*d.* $\frac{1}{2}$.

18. Calculate the interest of £145. 10*s.* for 5 yrs. 8 mths. at 3 p. c. per annum. *Ans.* £24. 14*s.* $8\frac{1}{2}$*d.*

19. What is the interest of £156. for 20 days, at 5 p. c. per annum? *Ans.* 8*s.* $6\frac{1}{2}$*d.* $\frac{11}{73}$.

20. Find the interest on £52. 12*s.* for 136 days, at 4 p. c. per annum. *Ans.* 15*s.* 8·149*d.*

21. Find the amount of £375. 13*s.* 4*d.* for 248 days, at $3\frac{1}{2}$ p. c. per annum. *Ans.* £384. 12*s.* 0·079*d.*

22. Lent £184. 12*s.* $9\frac{1}{4}$*d.* on the 12th March; what interest is due on the 5th August, at $2\frac{1}{2}$ p. c. per annum? *Ans.* £1. 16*s.* $11\frac{37}{400}$*d.*

23. What is the amount of £320. 11*s*. 10*d*., lent on the 17th May, and repaid on the 28th September, with interest at the rate of 3⅜ p. c. per annum?

Ans. £324. 11*s*. 3·343*d*.

24. Required the interest on £86. 14*s*. 6*d*., for 12 years, 156 days, at 5 p. c. per annum.

Ans. £53. 17*s*. 9·192*d*.

25. What principal will yield £43. 10*s*. 7½*d*. in 3 yrs. at 4 p. c. per annum? *Ans.* £362. 15*s*.

26. What principal, improved at simple interest, for 3 yrs. 11 mths., at 4½ p. c. per annum, will gain £100?

Ans. £567. 7*s*. 6·213*d*.

27. In what time will £230. gain £20. at 3 p. c. per annum? *Ans.* 2*yrs*. 328*da*.

28. At what rate p. c. per annum will £77. 10*s*. gain £7. 10*s*. in 3 yrs. 4 mths.? *Ans.* £2·903+.

29. What principal, improved for 7 yrs. at 2¾ p. c., will amount to £149. 1*s*. 3*d*.? *Ans.* £125.

30. A person borrows £260. at 3½ p. c., and at the same time £358. at 5 p. c. When will these loans require £640. 10*s*. to repay them together?

Ans. 10*mths*.

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2. Discount for Time.

Definitions.—Discount for time is an allowance deducted, to reduce money, due at some future time, to its Present Value.

The *true* Present Value of a debt due at some future time, is that sum which being now put out to interest will thereby suffice to pay the debt at the time it becomes due.

Bankers and merchants, in lending money, remunerate themselves *at the time of lending* by deducting from the sum borrowed what should be its interest *at the end of the term of credit*; and, accordingly, the Exercises in Bankers' Discount, may be considered as just a continuation of the Exercises in Simple Interest.

A knowledge of the principle of True Discount is of

service for the ascertainment of the real value of many pecuniary speculations.

Suppose I request a person to lend £100 for a year, the use of money being reckoned worth $4\frac{1}{6}$ per cent. per annum. The loan may be transacted in either of two ways, conformably to the true value of credit.

1. If the person charges me £4 *now*, he now gives me £96, which, if improved by me, at $4\frac{1}{6}$ p. c. per ann., will exactly enable me to pay what I owe at the year's end, viz. £100.

2. If the person charges me £$4\frac{1}{6}$ *at the year's end*, he now gives me £100, which, if improved by me, at $4\frac{1}{6}$ p. c. per ann., will exactly enable me to pay what I owe at the year's end, viz. £104. 3*s*. 4*d*.

It appears, then, that interest at $4\frac{1}{6}$ p. c. payable at a year's end, is equivalent to discount at 4 p. c. payable immediately.

But if, while the worth of money is $4\frac{1}{6}$ p. c. per ann., a person lending me £100 for a year, deducts a present remuneration of £4. 3*s*. 4*d*., he gives me £95. 16*s*. 8*d*., for which I have to pay him £100 at the year's end. So that the interest he has for his money is at the rate of £4. 3*s*. 4*d*. per ann. on £95. 16*s*. 8*d*., or about £4. 6*s*. $11\frac{1}{2}$*d*. p. c. per ann. He charges more than the true discount by the interest of that discount.

Bankers, &c. consider themselves entitled to charge in this way more than the common rate of interest, on account of the risks to which their capital is exposed.

To find the True Discount that will reduce money, due at some future period, to its present value, is to find—not the interest of the whole sum, but—the interest of the present value.

Examp. The sum of £345. is payable 3 years hence. What discount should be given as a compensation for paying it at present; the use of money being reckoned worth 5 p. c. per ann.?

Here, the Present Value, were it improved at 5 p. c. *per ann. for 3* years, should amount to £345; and, as

5 p. c. per ann. for 3 years is 15 p. c., we inquire, what sum increased by $\frac{15}{100}$ or $\frac{3}{20}$ of itself amounts to £345. Accordingly, $345 \div 1\frac{3}{20} = 6900 \div 23 =$ £300, the Pres. Val., or £45, the Disc. *Ans.*

The usual mode of solution, however, is by a Proportion statement, employing £100 as a given present value. Thus, in the question just resolved, there is given £100 as the present worth of £115; that is, £100 paid at present = £115 to be paid 3 yrs. hence. Accordingly, we say, If £100 is the present value of £115, what is the corresponding present value of £345?

£345 Future Worth,

115 : 100 :: £345 : £300 Present Worth,

Ans. £45 Discount.

Or, we may find the Discount by the direct inquiry, If £15 is the discount on £115, what is the corresponding discount on £345?

115 : 15 :: £345 : £45. *Ans.*

It is occasionally convenient to consider that the true discount of any sum which is due at a future period is that fraction of the sum which has the absolute rate per cent. for a numerator, and 100 + that rate for a denominator; and that the true present worth of any sum is that fraction of it which has 100 for a numerator, and 100 + the absolute rate for a denominator. Thus, in the above example, the discount is $\frac{15}{115} = \frac{3}{23}$ of £345; and the present worth is $\frac{100}{115}$ or $\frac{20}{23}$ of £345.

The two following examples will serve to illustrate the more difficult kinds of calculation that occur in the Rule of Discount.

Examp. If I purchase dresses at 10*s.* 3*d.*, with 6 months credit, and sell them at 12*s.* 9*d.*, with 3 months credit; what is my immediate gain p. c.?

Here, as no rate per cent. is mentioned, legal interest is to be understood. Accordingly, the use of money for 6 months is worth $2\frac{1}{2}$ p. c., and for 3 months $1\frac{1}{4}$ p. c. The present worths, therefore, of 10*s.* 3*d.* and 12*s.* 9*d.* are obtained by the following proportions:

$101\frac{1}{4}$: 100 :: 12*s.* 9*d.* : 12*s.* $7\frac{1}{2}$*d.* P. V. of 12*s.* 9*d.*

$102\frac{1}{2}$: 100 :: 10*s.* 3*d.* : 10*s.* 0*d.* P. V. of 10*s.* 3*d.*

The article costs me 10*s.* cash, and brings me 12*s.* 7½*d.* cash. On 10*s.*, therefore, my gain is 2*s.* 7½*d.*; and, to find my gain p. c.,

10*s.* : 2*s.* 7½*d.* :: 100 : 25$\frac{5}{17}$ p. c. *Ans.*

Examp. What should I charge, per yard, for cloth which stands me 19*s.* per English ell, that I may now have 25 p. c. profit, after allowing a discount of 5 p. c. and 6 months credit?

19*s.* an ell, with 25 p. c. profit, is 19*s.* a yard, which is to be the present worth of my receipts. Then, as 6 months credit is worth 2½ p. c.,

100 : 102½ :: 19*s.* : 19·475*s.* a yd., payable in 6 mo.
95 : 100 :: 19·475*s.* : 20*s.* 6*d.* a yd. must be charged, to allow 5 p. c. discount. *Ans.*

Exercises in Bankers' Discount.

1. I borrow from a bank the sum of £25. for 4 months. What sum will the banker retain as discount, at 5 p. c. per annum? *Ans.* 8*s.* 4*d.*

2. Required the discount on £164. 10*s.* for 9 mths. at 4½ p. c. per annum. *Ans.* £5. 11*s.* 0½*d.*

3. Find the reputed present worth of £452. 14*s.* due 3 yrs. 2 mths. hence, at 5 p. c. per annum.
Ans. £381. 0*s.* 5·4*d.*

4. What is the reputed present worth of £64. 16*s.* 6*d.* due 50 days hence, at 4 p. c. per annum?
Ans. £64. 9*s.* 4¾*d.*

5. A bill of £42. 15*s.* was due on the 3rd August, but was paid on the 10th June preceding. By what sum was it paid, reckoning the discount at 5 p. c. per annum? *Ans.* £42. 8*s.* 8*d.*

Exercises in True Discount.

1. Find the true present worth of £154. due 1 year hence, at 5 p. c. per annum. *Ans.* £146. 13*s.* 4*d.*

2. What is the proper discount on £124. payable in 2 years, at 4¼ p. c. per annum? *Ans.* £9. 14*s.* 3½*d.*

3. What is the correct, and what is the reputed, discount on £517. 10*s.* for 7 months, at 6 p. c. per annum?
Ans. Cor. £17. 10*s.*; *Rep.* £18. 2*s.* 3*d.*

4. Required the correct present worth of £85. for 5 yrs. at 5½ p. c. per annum? *Ans.* £66. 13*s.* 4*d.*

5. What is the true present value of £18. 10*s.* for 4½ yrs. at 4½ p. c. per annum? *Ans.* £15. 7*s.* $8\frac{1}{13}$*d.*

6. Find the true discount on £217. 15*s.* 6*d.* for 326 days, reckoning legal interest? *Ans.* £9. 6*s.* $2\frac{1}{4}\frac{3}{1}$*d.*

7. I owe £86., payable 5 months hence, and £95., payable 7 months hence. What sum paid at present should discharge both these debts, reckoning the use of money to be worth 4½ p. c. per annum?
Ans. £176. 19*s.* 8·93*d.*

8. A person who is salaried at £100. per annum, payable quarterly in arrears, wishes to know what sum, paid him at the beginning of the year, would be equivalent to that year's income—reckoning interest at 5 p. c. per annum. *Ans.* £96. 19*s.* 9*d.*

9. I bought a horse for 25 guineas, cash, and sold him the same day for 30 guineas, allowing 6 months' credit. What was my gain per cent? *Ans.* $17\frac{3}{7}\frac{3}{1}$*p.c.*

XIX. DIVISION INTO PROPORTIONAL PARTS.

When several numbers have each the same ratio to several other numbers, the ratio subsists also in the sum or difference of any corresponding terms in the two series.

Thus, let there be three numbers, 6, 9, and 4, and three other numbers, 36, 54, and 24, the latter being, respectively, 6 times the former; then, the sum of the three is, in the one case, 19, and in the

6 : 36	19 : 114
9 : 54	15 : 90
4 : 24	5 : 30
	7 : 42

other 6 times 19; the sum of the two first is 15 and 6 times 15; the difference of the second and third is 5 and 6 times 5; the sum of the second and third diminished by the first is 7 and 6 times 7; &c.

This persistency of the ratio is easily accounted for. In comparing the sum of *once* 6 and *once* 9, with the sum of *six times* 6 and *six times* 9, we must expect the sum of six times each of the numbers, to be six times the sum of once each of the numbers; and so of the other combinations, &c.

Now, a recognition of this persistency enables us easily to resolve a given amount into as many aggregate parts, having the same interproportion, as any given numbers. We are also enabled hereby to determine numbers whose difference and ratio are given.

Definition. DIVISION INTO PROPORTIONAL PARTS resolves a given amount into parts that have the same ratio, one to another, as certain given numbers have.

Examp. Divide £98. 2*s*. 2*d*. into four parts, proportioned as the numbers 12, 15, 16, and 18.

The first divided by 12, the second by 15, &c., are to yield the same quotient. Now, as the sum of all the required parts is given, we have to compare it with the sum of all the proportional parts, to determine the general ratio.

61 : 12 ::	£98. 2*s*. 2*d*.	:	£19	6*s*.	0*d*.	
— : 15 ::	————	:	24	2	6	*Ans.*
— : 16 ::	————	:	25	14	8	
— : 18 ::	————	:	28	19	0	

Otherwise, we might say, the first is 12 parts of the whole 61, or $\frac{12}{61}$ of the whole, the second $\frac{15}{61}$, &c. Then, as $\frac{1}{61}$ is £1. 12*s*. 2*d*., therefore, $\frac{12}{61}$ is £19. 6*s*.; and so on.

Examp. Divide £98. 2*s*. 2*d*. into four parts, which multiplied, respectively, by 12, 15, 16, and 18, shall give equal products.

The first divided by $\frac{1}{12}$, the second by $\frac{1}{15}$, &c., are

to yield the same quotient. Therefore, the four parts are to be proportioned as $\frac{1}{12}$, $\frac{1}{15}$, $\frac{1}{16}$, and $\frac{1}{18}$; or, multiplying these fractions by their least common denominator, 720, which will not alter their proportional relation, we obtain 60, 48, 45, and 40, correspondent to which the answers will be found, £30. 10*s.*, £24. 8*s.*, £22. 17*s.* 6*d.*, and £20. 6*s.* 8*d.*

Examp. Divide 84 guineas into three parts, so that for every 5 guineas in the first there may be 6 in the second, and for every 4 guineas in the second there may be 9 in the third.

The second is $\frac{6}{5}$ of the first, and the third is $\frac{9}{4}$ of the second, or $\frac{9}{4}$ of $\frac{6}{5}$ of the first, *i. e.* $\frac{27}{10}$ of the first. So that the three parts are as 1, $\frac{6}{5}$, and $\frac{27}{10}$, or as 10, 12, and 27. Dividing 84 guin. in the proportion of these numbers gives $17\frac{1}{7}$, $20\frac{4}{7}$, and $46\frac{2}{7}$, guin. *Ans.*

Examp. Three persons, A, B, and C, are to have a certain sum divided among them, in such a manner that $\frac{4}{5}$ of B's share shall be equal to $\frac{1}{2}$ of A's, and $\frac{1}{4}$ of A's to $\frac{1}{2}$ the sum of B's and C's; by this means the difference of the shares of B and C will be 10*s.* 6*d.* Required the whole sum, and each share.

B's share equals $\frac{5}{4}$ of $\frac{1}{2}$ of A's; also, $\frac{8}{5}$ of A's equals the sum of B's and C's; $\frac{8}{5}$ of A's $-$ $\frac{1}{8}$ of A's, or $\frac{59}{40}$ of A's, equals C's share; hence, A, B, and C, are as 1, $\frac{5}{8}$, and $\frac{59}{40}$; or as 40, 25, and 39.

Now B's and C's shares differing by 10*s.* 6*d.*, and their proportional parts by 39 $-$ 25, or 14, we have,

14	: 40	:: 10*s.* 6*d.*	: 30*s.*	0*d.*	to A.	}	*Ans.*
—	: 25	:: ———	: 18	9	to B.		
—	: 39	:: ———	: 29	3	to C.		
—	: 104	:: ———	: 78*s.*	0*d.*	in all.		

Sometimes the proportional parts require to be determined by compounding ratios, as in the following example:—

Examp. A merchant bequeaths £1000 among six clerks, in proportion to their salaries, and the periods they have held their situations. Now, one of them has held his situation 5 years, and his salary is £120: two of them 4 years, with salaries of £75 each; and the rest 2 years, with salaries of £60 each. Required their several shares.

120 × 5 = 600 the proportional part for one;
150 × 4 = 600 ditto for two others;
180 × 2 = 360 ditto for three remaining.

Accordingly £1000 divided in the proportion of these numbers, gives £384$\frac{8}{13}$ to one, £192$\frac{4}{13}$ to each of two, and £76$\frac{12}{13}$ to each of three.

EXERCISES.

1. Divide 72 into four parts, proportioned as the numbers 3, 4, 5, 6. *Ans.* 12, 16, 20, and 24.

2. Divide 3498 into three parts, so that the first divided by 7, the second by 6, and the third by 9, may give equal quotients. *Ans.* 1113, 954, and 1431.

3. Divide 3498 into three parts, so that the first multiplied by 7, the second by 6, and the third by 9, may give equal products. *Ans.* 1188, 1386, and 924.

4. Divide £47. 9*s.* into three parts, bearing the proportion of 8, 3, and 2.
Ans. £29. 4*s.*; £10. 19*s.*; £7. 6*s.*

5. There are three numbers proportioned as 17, 10, and 12$\frac{1}{2}$, and the sum of the second and third is 15. Required the numbers. *Ans.* 11$\frac{1}{4}$, 6$\frac{3}{4}$, 8$\frac{1}{4}$.

6. Four persons built a ship for £2607., whereof B. paid twice as much as A., C. as much as A. and B. together, and D. as much as B. and C. together. What did each pay? *Ans.* A. £237.; &c.

7. Divide £14. 8*s.* into two parts, so that the 15th part of the one may equal the 17th part of the other.
Ans. £6. 15*s.*; £7. 13*s.*

8. If James and I have 2*s.* 4*d.* between us, and my share is just $\frac{3}{17}$ of his; how much have we individually?
Ans. 1*s.* 10*d.* and 6*d.*

9. A. and C. have 5½*d.* together; A. has ¼ of B.'s money, and C. has ⅛ of A.'s. What has each?
Ans. A. 4*d.*; B. 10*d.*; C. 1½*d.*

10. If brass be composed of 63 parts of copper and 31 parts of zinc; what quantity of brass contains 4 lbs. more of copper than of zinc? *Ans.* 11*lbs.* 12*oz.*

11. Divide a burden of 2 cwt. 1 qr. 20 lbs. among 2 men, a woman, and a boy, so that the woman's burden may be half as great as each man's, and the boy's a third of the woman's. *Ans.* Each man 102*lbs.*; &c.

12. Divide 42 into two parts, so that the less divided by the greater may give $\frac{7}{17}$. *Ans.* 29¾ and 12¼.

13. Divide 39 into two parts, so that the greater divided by the less may give 39. *Ans.* 38·025 and ·975.

14. In an assemblage of 2090 persons, the number of women is ⅛ of the number of men, the boys ⅓ of the women, and the girls $\frac{3}{10}$ of the boys. How many persons are there of each description? *Ans.* 1800 *men*; &c.

15. What sum of money must be shared among A., B., and C., so that C. may have the 17th part of the whole, B. 17 times as much as A., and A. £17. less than B.? *Ans.* £20. 6*s.* 4$\frac{7}{17}$*d.*

XX. FELLOWSHIP.

Definition.—FELLOWSHIP is an application of the Rule of Proportional Division, to share a general profit or expense among commercial partners.

The Rule of Fellowship is so direct an application of the Rule of Proportional Parts, that the Exercises under the former title may be regarded as a continuation of those under the latter.

We shall here, however, give some illustration of Compound Fellowship, or Fellowship with Time.

Three things are principally to be considered in Compound Fellowship,—the capital formed by the stocks of the several partners, the respective periods of the continuance of these stocks in trade, and the respective gains or losses on the stocks.

The relations of these three particulars are the following:—

The Gains (or the Losses) are as the products of Stock and Time. The Stocks are as the quotients of Gain by Time. The Times are as the quotients of Gain by Stock.

Examp. A, B, and C, are mercantile partners: A's stock being £540, which he continues for 5 months; B's £460, for 3 months; C's £560, for 6 months. They gain by the entire capital, £248. Required each partner's share of this gain.

Here, £540 gains in 5 months what £540 × 5 would gain in 1 month; also £460 for 3 months = £460 × 3 for 1 month; and £560 for 6 months = £560 × 6 for 1 month. The gains of A, B, and C, therefore, are as 2700, 1380, and 3360.

7440	: 2700	::	£248	: £90, A's gain.	} *Ans.*
——	: 1380	::	——	: 46, B's do.	
——	: 3360	::	——	: 112, C's do.	
	£7440			£248	

Examp. Suppose the data to be, each of the above gains, and each of the times, together with the difference of the stocks of A and B = £80; to find each stock.

A's gain per month is £90 ÷ 5, B's £46 ÷ 3, C's £112 ÷ 6; therefore, their stocks are as 18, $15\frac{1}{3}$, and $18\frac{2}{3}$, or as 54, 46 and 56.

54 − 46	: 54	::	£80	: £540, A's stock.	} *Ans.*
——	: 46	::	—	: 460, B's do.	
——	: 56	::	—	: 560, C's do.	

Examp. Suppose, now, the data to be, each of the gains, and each of the stocks, together with the sum of

the times = 14 months; to find the time of each investment.

A gains $\frac{80}{340}$ of his stock, B $\frac{48}{480}$, and C $\frac{112}{560}$; their times must, therefore, be proportioned as $\frac{1}{6}$, $\frac{1}{10}$, and $\frac{1}{5}$; and if 14 months be divided according to these proportional parts, we obtain A's time 5, B's 3, C's 6, months. *Ans.*

EXERCISES.

1. Two partners gained in one year £517. 12*s*. 6*d*. Divide that sum between them according to their shares of capital, which are £1400. and £1100.

Ans. £289. 17*s*. 4·8*d*.; £227. 15*s*. 1·2*d*.

2. A., B., and C. are partners in business, and contribute, respectively, £350., £400., and £420., to form the joint capital. They gain £270. What is each party's share in that profit? *Ans.* A. £80. 15*s*. $4\frac{8}{13}$*d*.; &c.

3. A ship was purchased for £1200., by A., B., and C.,—the respective advances being £465. 10*s*., £376. 10*s*., and £358. They now sell the vessel for £800. What does each receive of the proceeds of sale?

Ans. A. £310. 6*s*. 8*d*.; &c.

4. The capital in a mercantile speculation is divided into 50 shares, of which A. holds 8, and B. 5. Required the respective claims of those parties on a general profit of £175. *Ans.* £28. and £17. 10*s*.

5. D., E., F. and G. form a co-partnery, with a joint stock of £3850.; D.'s investment being $\frac{1}{5}$ of that sum, E.'s $\frac{1}{7}$, and F.'s £700. They incur a loss of £154.;—to what amount is that loss sustained by each party?

Ans. D. £30. 16*s*.; &c.

6. A., B., and C. enter into partnership, with a capital of £7000.: A.'s share is £4000., B.'s £1800., and C.'s £1200. A., being manager, is allowed, besides his share of the profits, £150. of salary. At the end of the year they find the amount of their gains to be £1074. Divide that sum among them.

Ans. A. £678., B. £237. 12*s*., C. £158. 8*s*.

7. A., B., and C. join stocks in trade. A. puts in £530. for 10 months, B £480. for 1 year, and C. £556. for 15 months. They lose £271. 12*s*. Required the particular shares of loss. *Ans.* A. £74. 4*s*.; &c.

8. A. and B. rent a field of grass for £61.; A. puts in 27 oxen for 75 days, and B. 56 oxen for 39 days. How much of the rent should be paid by each?

Ans. A. £29. 6*s*. $11\frac{1}{4}d.\frac{2}{3}\frac{1}{3}$; &c.

9. The original owners of a steam-vessel having sold out their shares at different times, the present proprietors, A., B., and C., agree to share a profit of £170., in proportion to the time each has been concerned in the vessel, and the sum he has advanced. A has advanced £620. for 14 months, B. £490. for 15 months, and C. £420. for 10 months. Required the particular shares of profit? *Ans.* A. £72. 18*s*. $9\frac{1}{7}d.$; &c.

10. A. and B. rent a grass enclosure for £35.;—A. puts in 120 sheep for 5 months; and B. puts in 6 horses for 4 months, and 7 oxen for 6 months. What should each pay of the rent,—supposing a horse to eat as much in a day as 16 sheep, and half as much again as an ox?

Ans. A. £14. 13*s*. $3\frac{8}{17}\frac{9}{5}d.$; B. £20. 6*s*. $8\frac{8}{17}\frac{9}{5}d.$

XXI. SQUARE AND CUBIC MEASURE.

A Square is a surface contained by four sides, equal to each other, and joined perpendicularly.

Square Measure is so called from its representing the number of times that a certain square unit is contained in a given surface or area.

The Exercises in Square Measure have reference only to quadrilateral surfaces, with perpendicular sides. And of such figures it is easily demonstrable, that the number of lineal units in any one of the sides multiplied *by the number* of lineal units in a side that is perpen-

dicular to that one, yields the number of square units in the whole surface.

Suppose the length is 8 inches, and the breadth 3 inches, then, 8 × 3, or 24 square inches, is the surface measure of the figure.

Definition.—SQUARE or SURFACE MEASURE is the Product of *Length* and *Breadth.*

A Cube is a solid contained by six superficial sides, equal to each other, and joined perpendicularly.

Cubic Measure is so called from its representing the number of times that a certain cubical unit is contained in a given solid or volume.

As the product of the number of units in length by the number of units in breadth, is the number of square units in one superficial side, so it will represent also the number of cubical units for 1 unit of depth; and, therefore, the product of the units of length and breadth, multiplied by those of depth or thickness, will give solid measure.

Definition.—CUBIC or SOLID MEASURE is the Product of *Length*, *Breadth*, and *Depth.*

The Exercises in Square and Cubic Measure are so elementary as to render an exhibition of examples here unnecessary. Only, it must be remembered, that in performing these Exercises, the dimensions must be applied to each other under a uniform denomination. Thus, 3 ft. 3in. × 2 ft. 11 in. $= 3\frac{1}{4}$ ft. × $2\frac{1}{2}$ ft. $=$ $(13 \times 35) \div 48 = 9$ sq. ft. 69 sq. in.

EXERCISES.

1. How many square yards are in a room floor, whose length is 14 ft. and width $11\frac{1}{2}$ ft.? *Ans.* $17\frac{5}{9}$.

2. Required the area of a court 13 ft. wide and 15 ft. 4 in. long. *Ans. 22sq.yds. 1ft. 48in.*

3. Find the area of a square, each of whose sides is 137 ft. *Ans.* 2085sq.yds. 4ft.

4. What is the surface measure of a board 3 ft. 3 in. by 2 ft. 11 in.? *Ans.* 9*sq.ft.* 69*in.*

5. The area of a room is 387 sq. ft.,—its breadth being 18 ft. What is its length? *Ans.* 21½*ft.*

6. An oblong surface contains 22 sq. in.,—its length being ·408 of a foot. Required its breadth.

Ans. 4·4935*in.*

7. What length, with 5½ inches breadth, will make a square foot? *Ans.* 2*ft.* 3*in.*

---

8. What is the solidity of a block of stone, 4 ft. long, 2½ broad, and 1¾ ft. thick? *Ans.* 17½*cub.ft.*

9. What is the solidity of a cube, each of whose sides is 16 in.? *Ans.* 2½⅞*sol.ft.*

10. How much air in a room 15 ft. long, 13 ft. 8 in. wide, and 10 ft. 10 in. high? *Ans.* 82·253*cub.yds.*

11. What is the solidity of a cube of ivory $\frac{5}{16}$ of an inch in breadth? *Ans.* ·091125*c.in.*

12. The area of a ceiling is 105 sq. ft. 65 sq. in., and its height from the floor is 11⅓ ft. How many cub. ft. of air does the room contain? *Ans.* 1181*c.ft.* 96*in.*

13. A stone roller, each of whose ends measures 162 sq. in., contains 3 solid feet. What is its length?

Ans. 2*ft.* 8*in.*

XXII. SQUARE ROOT.

Products considered as resulting from the continual multiplication of a number by itself, are called *powers* of that number. Thus 8 is a power of 2, because 2 × 2 × 2 = 8; also, 30¼ is a power of 5½, because 5½ × 5½ = 30¼. The number continually multiplied is called the *root* of the power, and is said to be *involved* or *raised* to the power.

A power of a number is denoted by a small figure, called the *index*, because it *indicates* how often the root is used as a factor; thus, $4^2 = 16$ means $4 \times 4 = 16$, or that the second power of 4 is 16; $2^3 = 8$ means $2 \times 2 \times 2 = 8$, or that the 3rd power of 2 is 8.

The 2nd power of a number is usually called the *square* of that number, and the 3rd power of it is called its *cube*. But, properly, the terms square and cube are names of *geometrical*, not *arithmetical*, quantities.

The process of finding any root of a number is called *Evolution*, or *extracting the root.*

The sign $\surd$ (a form of the initial letter of *radix, i. e. root*), when prefixed with an index figure to a number, denotes evolution; thus, $\sqrt[4]{625} = 5$ means that the 4th root of 625 is 5; $\sqrt[3]{729} = 9$ means that the cube root of 729 is 9. When no index figure is used, the square root is meant, as $\surd 625 = 25$.

Definition.—The SQUARE ROOT of a number is one of the two equal factors which produce that number.

When a square number is given whose root does not exceed 12, a recollection of the Multiplication Table enables us to tell the root at once. But, with larger numbers, and such as are not exact squares, the extraction of the root is performed by successive steps suggested by a particular method of involution.*

In proceeding to illustrate this, it may be observed, that if any number be divided into two parts, the square of the whole number may be constructed by means of the two parts, thus; square the first part, and then add the product which arises from multiplying the sum of the second and twice the first parts by the second part.

Suppose, for example, the square of 39 is to be constructed by means of the aggregate parts 30 and 9;

Placing the one number over the other $\begin{matrix} 30 + 9 \\ 30 + 9 \end{matrix}$

The top line $\times$ 30 gives { 30 times 30
9 times 30

The top line $\times$ 9 gives { 9 times 30
9 times 9

* That which represents $(a + b)^2$ by the form $a^2 + (2a + b) \times b$; corresponding to Euclid, Book II. Prop. 6.

Hence it appears that 39 × 39, or 1521, consists of 30 times 30 + 9 times (30 + 30 + 9), that is, $39^2 = 30^2 + (2ce\ 30 + 9) \times 9$.

In this manner the square of any larger number may be constructed. For example: to construct by this method the square of 273; first find 270^2 by means of the two parts 200+70, and then the square of 273, or 74529, by means of the two parts 270 + 3.

Thus $270^2 = 200^2 + (2ce\ 200 + 70) \times 70$; also $273^2 = 270^2 + (2ce\ 270 + 3) \times 3 = 200^2 + (2ce\ 200 + 70) \times 70 + (2ce\ 270 + 3) \times 3$.

The advantage of this method of expressing a square number is, that it introduces the hundreds, tens, &c. of the root successively, and thus furnishes an easy rule for the extraction of the square root.

The first thing to be done in proceeding to extract the square root of a large number, is to ascertain the rank of the first or left-hand figure of the root. For this purpose, point off the left-hand figure of the square, if the whole number of figures is odd, otherwise, point off two figures; the square of the first or left-hand figure of the root is contained in what has been thus pointed off, and the remaining number of figures in the root will be one for every two remaining figures of the square. Thus for the square number 5798464, point off the left-hand figure 5, which contains the square of the left-hand figure of the root, and there remain three periods of two figures each; therefore, the whole number of figures in the root must be four, or the highest denomination in the root is thousands. Thus also for the square number 483025, point off the two figures 48, which contain the square of the left-hand figure of the root, and there remain two periods of two figures each; so that the root must consist of three figures.

The reason of this method of ascertaining the number of figures in the root will be manifest, if we consider that in squaring a number consisting of one significant figure followed by ciphers, we shall have a product consisting of one, or at most two, figures, followed by two ciphers for every cipher of the root; thus, 300 × 300 = 9,00,00; also 7000 × 7000 = 49,00,00,00. For although, when significant figures are substituted for the ciphers, as in the example 365 × 365, the left-hand period of the square may thereby become increased to two figures, there will be no greater number of periods in the square. As 100 × 100 gives 1,00,00, the smallest square consisting of three periods, so 1000 × 1000 gives 1,00,00,00, the smallest square consisting of more than three periods.

Let us now exemplify the process of finding *successively the* figures of the root of a given square.

Examp. Required the square root of 74529.

$\sqrt{7,45,29} = 200 + 70 + 3$, or 273. *Ans.*

	4 00 00	
400	3 45 29	rem. after subtracting 200^2.
70		
470	3 29 00	$= (2ce\ 200 + 70) \times 70$.
540	16 29	rem. after subtracting 270^2.
3		
543	16 29	$= (2ce\ 270 + 3) \times 3$.

0 rem. after subtracting 273^2.

The given square, in the above example, consists of three periods, and, therefore, the first part of the root is hundreds. Now, the highest square contained in 70000 is that of 200, viz. 40000; therefore, subtracting 200^2, we have to find what the tens of the root must be, so that (2*ce* 200 + *the tens*) × *the tens* may give the highest subtrahend for 34529. For this purpose, we use 2*ce* 200 as a *partial* divisor of that number, in which it goes 80 times; but, making trial of 80 times, we find that the *complete* divisor 480, if taken 80 times, would exceed the dividend; therefore, trying 70 times, we find that to be the proper number, forming the complete divisor 470. Lastly, we have to find what the simple units of the root must be, so that (2*ce* 270 + *the units*) × *the units* may give the highest subtrahend for 1629. For this purpose we use 2*ce* 270 as a partial divisor of 1629, in which it goes 3 times; and making trial of 3 times, we find that the complete divisor 543, if taken 3 times, exactly equals the dividend. Accordingly, 273 is the exact root of the given square.

The above process may be abbreviated by suppressing the ciphers. The following, therefore, is the usual method of extracting the square root.

RULE. Beginning at the place of units, mark off, from right to left, as many periods of two figures each as the given number allows. Mark off decimals (if any) from left to right. Under the left-hand period write the greatest square number it contains, and *place the root* of that number in the

	7,45,29 (273 *Ans.*
	4
47	345
7	329
543	1629
	1629

quotient. Subtract the square number from the left-hand period; to the remainder annex the next period for a dividend; and, for its trial divisor, double the quotient figure. Consider what is the greatest number of times which, multiplied by the sum of that number and ten times the trial divisor, will yield a number contained in the dividend; this will supply a second quotient figure, which, being annexed to the trial divisor, forms the complete divisor. Multiply the complete divisor by the quotient figure; subtract the product from the dividend; to the remainder annex the next period, for a new dividend; and, for a new trial divisor, either double the quotient, or add the last quotient figure to the last complete divisor. (When the partial divisor gives 0 as a quotient figure, annex 0 to that divisor, bring down the next period, and renew the trial.) Continue the operation till all the figures in the given number have been used; and, if there be any remainder, annex periods of two ciphers to find a decimal continuation of the root to the extent required.

Most numbers are not exact squares; but we can always obtain the root with a degree of accuracy sufficient for practical purposes. In the two following examples the exact roots are interminate quantities.

Examp. Evolve $\surd 4809{\cdot}523$, and $\surd {\cdot}006$.

```
      48,09·52,30(69·3507 +.   Ans.
      36
      --
129 | 1209
    | 1161
      ----
1383 |  4852
     |  4149
       -----
13865 |  70330
      |  69325
        ------
1387007 | 10050000
```

```
      ·00,60(·07746 — .
         49          Ans.
         --
147 | 1100
    | 1029
      ----
1544 |  7100
     |  6176
       -----
1548 |  92400
```

When it is proposed to extract the square root of a vulgar fraction, evolve numerator and denominator separately, if they are exact squares; thus, $\surd \frac{1}{4} = \frac{\surd 1}{\surd 4} = \frac{1}{2}$; also, $\surd 1\frac{7}{9} = \frac{\surd 16}{\surd 9} = \frac{4}{3} = 1\frac{1}{3}$. Otherwise, reduce to the decimal form, and then extract; thus, $\surd \frac{3}{8} = \surd {\cdot}375 = {\cdot}61237$; or first multiply both *terms of the fraction* by the denominator, making $\surd \frac{3}{8} = \frac{\surd 24}{\surd 64}$ $= 4{\cdot}89898 \div 8 = {\cdot}61237$.

It may be observed, that, as the successive divisors in evolution tend more and more to uniformity in their left-hand figures, we might, after proceeding a little way with the usual operation, obtain additional figures by means of one divisor, working according to the contracted form of decimal division. Thus, in the second example, the last dividend when divided by 1387007, will yield the additional figures of the more accurate root, 69·35072458.

EXERCISES.

Required the square roots of (1) 841; (2) 2209; (3) 86436; (4) 323761; (5) 94249; (6) 39363076; (7) 348100; (8) 99; (9) 2; (10) 3; (11) 3139; (12) 222; (13) 47·3; (14) 25625·6064; (15) ·6; (16) 1076·5; (17) ·07; (18) ·001; (19) 8101·079; (20) $\frac{3}{4}\frac{6}{6}$; (21) $\frac{6}{11}\frac{8}{2}$; (22) $\frac{6}{7}$; (23) $\frac{5}{9}$; (24) $30\frac{1}{4}$; (25) $38\frac{4}{7}$.

(26) A room is 10 ft. 4 in. long and 9 ft. 3 in. wide; required the length and width of a square room which is one third larger.

Answers. (1) 29; (2) 47; (3) 294; (4) 569; (5) 307; (6) 6274; (7) 590; (8) 9·94987 +; (9) 1·41421 +; (10) 1·73205 +; (11) 56·02678 −; (12) 14·89966 +; (13) 6·8775 −; (14) 160·08; (15) ·7746 −; (16) 32·81006 −; (17) ·26457 +; (18) ·0316 +; (19) 90·0059 +; (20) $\frac{6}{7}$; (21) $\frac{3}{4}$; (22) ·92582 +; (23) ·745356 −; (24) $5\frac{1}{2}$; (25) 6·21059 +; (26) 11·289 *ft.*

XXIII. CUBE ROOT.

Definition.—The CUBE ROOT of a number is one of the three equal Factors which produce that number.

In beginning to learn the extraction of the cube root, the pupil should become able to name readily the cubes of the nine digits, viz., 1, 8, 27, 64, &c.

For the evolution of the cube root of larger numbers, we employ, as in the square root, a process suggested by a particular method of involution.*

The cube of any number may be constructed by means of two aggregate parts, thus; cube the first part, and add to it the product which arises from multiplying by the second part the following quantity; thrice the square of the first part + the sum of the second and thrice the first parts multiplied by the second part.

Suppose, for example, the cube of 540 is to be constructed by means of the aggregate parts 500 and 40.

The square is $500^2 + (2ce\ 500 + 40) \times 40$.
Multiply by $500 + 40$

The top line, × 500 and × 40, gives respectively,
$500^3 + 2ce\ 500^2 \times 40 + 500 \times 40 \times 40$,
and $500^2 \times 40 + (2ce\ 500 + 40) \times 40 \times 40$,

$500^3 + 3ce\ 500^2 \times 40 + (3ce\ 500 + 40) \times 40 \times 40$,
the same as
$500^3 + [3ce\ 500^2 + (3ce\ 500 + 40) \times 40] \times 40$.

Similarly, the cube of 546, $= (540 + 6)^3$, would be the above expression increased by $[3ce\ 540^2 + (3ce\ 540 + 6) \times 6] \times 6$.

This form of involution furnishes the usual Rule for the extraction of the cube root.

Now, the first thing to be done in proceeding to extract the cube root of a large number, is to ascertain the rank of the first or left-hand figure of the root. And, by a consideration similar to that exemplified under the square root, we find that, by pointing off a cube number into periods of three places, we shall have on the left-hand the figures containing the cube of the first figure of the root; and the remaining figures of the root will be one for every three remaining figures of the cube.

We shall now exemplify the process of finding successively the figures of the root of a given cube.

* Namely, that which represents $(a + b)^3$ by the form $a^3 + [3a^2 + (3a + b) \times b] \times b$.

Examp. Find the cube root of 162771336.

Ans. $546 = \sqrt[3]{162,771,336} = 500+40+6.$

	125 000 000
$3ce\ 500^2 = 750000$	37 771 336
$(3ce\ 500+40)\times 40 = 61600$	
811600	32 464 000
$3ce\ 540^2 = 874800$	5 307 336
$(3ce\ 540+6)\times 6 = 9756$	
884556	5 307 336

In the above example, the cube consists of three periods, and therefore the first part of the root is hundreds. The highest cube in 162000000 is that of 500. viz. 125000000, which being subtracted, we have to find what the tens of the root must be, so that $3ce\ 500^2+(3ce\ 500 + \textit{the tens}) \times \textit{the tens}$ may give the highest subtrahend for 37771336. For this purpose we use $3ce\ 500^2$ as a partial divisor of that number, in which it goes 50 times; but, allowing for the completion of the divisor, it goes only 40 times. The remainder of the process is similar to that exemplified in the extraction of the square root.

The above form of evolution may be abbreviated by suppressing ciphers. But, first, we may notice a very convenient method of finding the second trial divisor. In the given example we may readily determine $3ce\ 540^2$, by adding 40^2 or 1600 to the complete divisor 811600 and the number above it, for 61600 + 811600 + 1600 = 874800.* Subsequent trial divisors would admit the same method. Accordingly the general Rule for the extraction of the Cube Root may be stated as follows:—

RULE. Beginning at the place of units, mark off, from right to left, as many periods of three figures each as the given number allows. Mark off decimals (if any) from left to right. Under the left-hand period write the greatest cube number it contains, and place the root of that number in the quotient. Subtract the cube number from the left-hand period; to the remainder annex the next period for a dividend; and, for its trial divisor, take thrice the square of ten times the quotient figure. Find by trial

* The above method of finding $3\ (a + b)^2$ regards it as $= 3ce\ a^2 + (6\ a + 2\ b) \times b + b^2$, which is $= 3\ a^2 + (6\ a + 3b) \times b = 3ce\ [a^2 + (2\ a + b) \times b]$.

the second quotient figure; and to form the complete divisor, annex the second quotient figure to thrice the first, and multiply by the second, adding the product to the trial divisor. Multiply this complete divisor by the second quotient figure, which should give the highest attainable number for subtraction from the dividend. To find the next trial divisor add together the square of the second quotient figure, the last complete divisor, and the number standing above that divisor, and to the sum of these three annex two ciphers. Then, having by means of this new trial divisor ascertained the third quotient figure, annex that figure to thrice the preceding part of the quotient, and then multiply by that figure, adding the product to the trial divisor to form the complete divisor. (When the trial divisor gives 0 as a quotient figure, annex two ciphers to that divisor, bring down the next period, and renew the trial.) Continue the operation till all the figures in the given number have been used; and, if there be any remainder, annex periods of three ciphers to find a decimal continuation of the root to the extent required.

According to this Rule, the extraction of the cube root of 162771336 will assume the subjoined form.

$50^2 \times 3 =$	7500	162,771,336(546
$154 \times 4 =$	616	125
	8116	37771
$4^2 =$	16	32464
	874800	5307336
$1626 \times 6 =$	9756	5307336
	884556	

EXERCISES.

Required the cube roots of (1) 117649; (2) 438976; (3) 182284263; (4) 521660125; (5) 888057908928; (6) 263128269312; (7) 2; (8) 100; (9) 126·09; (10) ·01; (11) $\frac{27}{125}$; (12) $\frac{2}{3}$; (13) $\frac{1}{216}$; (14) $3\frac{3}{5}$; (15) $64\frac{1}{3}$.

(16.) A block of wood is 1·96 yds. long, 18·2 in. broad, and ·84 ft. thick; find the length of a cubical portion of it $= \frac{1}{1000}$ of the whole.

Answers. (1) 49; (2) 76; (3) 567; (4) 805; (5) 9612; (6) 6408; (7) 1·25992+; (8) 4·64159−; (9) 5·01449+; (10) ·21544+; (11) $\frac{4}{5}$; (12) ·87358+; (13) .·32549−; (14) $1\frac{1}{2}$; (15) 4·00416+; (16) 2·348−*in.*

XXIV. MISCELLANEOUS QUESTIONS.

1. Three quarters of wheat are worth 56*s.*, 59*s.*, and 61*s.*, respectively. Required the average value.

Ans. 58*s.* 8*d.*

2. Spain and Portugal together occupy 216000 square miles, the area of P. being a fifth of the area of S. How many acres in each? *Ans.* P. 23040000*ac.*; &c.

3. John's farm would be 9 times as large as it now is, if he had 42 acres more. How much land has he?

Ans. 5*ac.* 1*ro.*

4. A father was 32 years old when his daughter was born, and his age is now thrice hers. How old is he?

Ans. 48.

5. T. and H. dig a trench 372 yards in length, working towards each other. T. digs 26 yds. per day, and H. 36. How soon will they finish it? *Ans.* 6*da.*

6. A quantity of cloth at 3*s.* 6*d.* the yd., and an equal quantity at 3*s.* $5\frac{1}{4}$*d.* the E. ell, cost me together £8. 15*s.* How many yds. of each? *Ans.* 28.

7. A gold watch and guard-chain cost £20.,—the watch being £11. 10*s.* dearer than the chain. Find the price of each article. *Ans.* Watch £15. 15*s.*; &c.

8. I have 2 gall. of spirits worth 18*s.* 6*d.* a gall., 3 at 19*s.* 10*d.*, and 5 at 20*s.* What is the average value per gall.? *Ans.* 19*s.* $7\frac{1}{2}$*d.*

9. Multiply a certain number by 3, and subtract 7 from the product, and you have left 4 score. What is the number? Ans. 29.

10. Divide a certain number by 8, and add 46 to the quotient, and you obtain 3 score. What is the number? *Ans.* 112.

11. Add 36 to a certain number, and multiply the amount by 3, and you obtain the fifth part of 4680. What is the number? *Ans.* 276.

12. Multiply a certain number by 5, add 17 to the product, divide the amount by 6, and the result is 47. What is the number? *Ans.* 53.

13. The year 1845 commenced on a Wednesday. Required the day of the month for the first Saturday of August in that year? *Ans.* Aug. 2nd.

14. The year 1847 began on a Friday. What day of the week corresponds to the 18th of June in that year? *Ans.* Friday.

15. From Friday, Sept. 6, 1844, find the day of the week on which that year began. *Ans.* Monday.

16. Divide £97. 16*s.* 3*d.* into three equal numbers, of guin., crowns, and pence, respectively. *Ans.* 75 of each.

17. Divide 386 days into three equal numbers, of weeks, days, and hours, respectively. *Ans.* 48 of each.

18. Twelve men are occupied in a work which they can accomplish in 24 days; but, after they have wrought 6 days, 5 of them are withdrawn. In how many days thereafter should the work be completed by the remaining workmen? *Ans.* 30$\frac{4}{7}$*da.*

19. Find two numbers whose difference is 13$\frac{1}{2}$, the quotient of the greater $\div$ the less being also 13$\frac{1}{2}$. *Ans.* 14·58 and 1·08.

20. Fifteen men are occupied in a work which they can accomplish in 18 days; after they have wrought 8 days, how many men should be added to their number, so that the remainder of the work may require only 6 days? *Ans.* 10*m.*

21. Divide a school of 100 boys into five classes, so that each superior class shall contain 2 boys fewer than the next inferior. *Ans.* 16, 18, 20; &c.

22. If a square field measures 1 acre; what is the *length of its side*? *Ans.* 69·57 + *yds.*

23. An empty water-cistern, whose capacity is 60 gallons, has its discharging and supplying pipes set open at once,—the former running as much in 5 hours, as the latter does in 2 hours 20 minutes. How much water is in the cistern, when the supplying pipe has run 60 gallons? *Ans.* 32 *gall.*

24. Bought quills at 4*s.* 7*d.* the hundred, and sold them so as to gain $\frac{1}{8}$ of the selling price. Find the selling price. *Ans.* 7*s.* 4*d.*

25. An oblong apartment, 18 ft. longer than broad, measures 104 ft. in circuit. Find its length and breadth. *Ans.* 35*ft.* by 17*ft.*

26. R. lends T. £182. for 60 days. T. on another occasion lends R. £65. for 70 days. The party still under obligation proposes to grant the other a loan of £35. for such a time as shall cancel the obligation. What time must that be? *Ans.* 182*da.*

27. Bought cinnamon at 7*s.* a lb., and sold half of it at 6*d.* an oz. At what price per oz. may the other half be sold, that the gain on the whole may be 10 p. c.? *Ans.* 5$\frac{1}{2}$*d.*$\frac{1}{3}$.

28. If £40. 15*s.* and £57. 12*s.* 9*d.*, due at present, are paid 223 days, and 2 yrs. 73 da. hence, respectively; how much interest, at 5 per cent. per annum, should be given with each payment? *Ans.* £1. 4*s.* 10$\frac{3}{4}$*d.* and £6. 6*s.* 9$\frac{1}{4}$*d.*

29. G. and S., when 36$\frac{1}{2}$ miles apart, commence travelling towards each other—G. going faster by 528 yds. an hour. They meet in 5 hours. Find the rates of travelling. *Ans.* G. 3$\frac{1}{4}$*mi.*, S. 3$\frac{1}{4}$*mi.*, *an hr.*

30. I am now 40 years old, and my son's age is 9; in what time will his age be a third of mine? *Ans.* 6$\frac{1}{2}$*yrs.*

31. How much cloth shall I have for £274. 2*s.* 8$\frac{1}{2}$*d.* if the yds. I get be the same in number as the pence which I give for each yd.? *Ans.* 256$\frac{1}{2}$*yds.*

32. A. sells goods to B., and gains 4$\frac{3}{4}$ p. c.; B. then sells them to C. for £150., and gains 3$\frac{1}{2}$ p. c. What did the goods cost A.? *Ans.* £138. 9*s.* 3·79*d.*

33. The sum of £46. has been lying at interest for the last 18 months, at $3\frac{1}{2}$ p. c. per ann. If the amount is now divided among A., B., and C., giving B. 3*s.* 6*d.* more than A., and C. 4 times as much as B.; what will each receive? *Ans.* A. £7. 18*s.* $5\frac{3}{5}d.$, &c.

34. If 3 peasants, or 4 soldiers, can excavate 21 cubic yards of earth, for an entrenchment, in a day of $8\frac{1}{2}$ hours; in what time should 7 peasants and 9 soldiers excavate 64 cub. yds., supposing the soil, in the latter case, less favourable to expertness by one-fifth? *Ans.* $6\frac{2}{3}hrs.$

35. There are three vessels, C., D., and E.:—C.'s capacity is $\frac{4}{5}$ of D.'s; and E.'s capacity is $\frac{3}{8}$ of C.'s, and $1\frac{1}{2}$ gallon less than D.'s. Required the capacity of each. *Ans.* C. $1\frac{7}{7}$, D. $1\frac{3}{7}$, E. $\frac{9}{56}$, *gall.*

36. Divide 85 into two parts, so that $\frac{2}{3}$ of the one may be $\frac{4}{5}$ of the other. *Ans.* 45 and 40.

37. Divide 5*s.* $6\frac{3}{4}d.$ into 4 parts, so that the first may be $\frac{8}{10}$ of the fourth, the third $\frac{8}{10}$ of the first, and the second the sum of the first and third. *Ans.* $11\frac{1}{4}d.$, &c.

38. Divide £79. 18*s.* 4*d.* among 4 men, 7 women, and 11 boys, so that each woman may have 3 boy's shares, and each man 4 woman's shares.

Ans. Each boy 19*s.* $11\frac{3}{4}d.$, &c.

39. A contractor has so many workmen, that a certain work which he has undertaken can be accomplished by them in 52 days; and, if he had 63 more men, the same work could be accomplished in 40 days. How many men has he? *Ans.* 210*m.*

40. If bread, when at a certain price, costs a certain company a certain sum, for a certain time; how much has the bread risen or decreased in price, when $\frac{1}{5}$ more of a company, consuming half as rapidly, are charged $\frac{1}{7}$ of the sum, for $\frac{1}{4}$ of the time? *Ans. Risen, as* 21 : 25.

41. There are two towns, A. and B., 26 miles apart, and a third town, C., in a straight line between them, 2 miles farther from A. than from B. Where must a point be placed, to mark off their proportional distances on a line 3 feet long? *Ans.* $19\frac{5}{13}$*in. from the end.*

42. There are three numbers whose sum is 70; the second divided by the first yields 2 as quotient, and 1 as remainder; and the third divided by the second yields 3 as quotient, and 3 remainder. Required the numbers.
Ans. 7, 15, 48.

43. The imperial gallon contains 277$\frac{1}{4}$ cubic inches: What must be the depth of a cubical cistern to hold 160 gallons of water? *Ans.* 35·4*in.*

44. What is the prime cost of one yard of cloth, if selling it at 7 p. c. advance bring 5*s.* more than selling it at 17 p. c. loss? *Ans.* £1. 0*s.* 10*d.*

45. There are 1200 men employed in a work which they can complete in 12 weeks; but, at the end of 2 weeks, 300 workmen are added, and, 2 weeks after that, 500 workmen are withdrawn. How long should the work last? *Ans.* 13*wks.*

46. A. and B. have a joint capital of £1000.:—A.'s stock remains 9 months, and his share of gain is £90.; B.'s time is 6 months, and his gain £65. Required the particular stocks. *Ans.* A. £480., &c.

47. The joint capital of two partners (whose particular stocks differed by £100.) was to the greater share as 14 : 9. Required each stock. *Ans.* £225. and £125.

48. If I sell to one person 4 horses and 10 cows for £165. 6*s.* 8*d.*, and to another, at the same prices, 6 horses and 7 cows for the same amount; what do I receive for one of each? *Ans.* H. £15. 10*s.*; &c.

49. A. and B. enter into partnership, A.'s stock being £103. more than B.'s: A.'s time is 11 months, and his gain £90.; B.'s time is 15 months, and his gain £104. Required each stock. *Ans.* A. £675.; &c.

50. Bought wheat at 61*s.* a quarter, payable in 4 months, and sold it, the same day, at 65*s.* payable in 7 months. Required the immediate gain per cent.
Ans. 5$\frac{5}{11}$*p.c.*

51. If a certain number be multiplied by 5, the ninth part of the product, diminished by 22, leaves one-fourth of the number. Required the number. *Ans.* 72.

52. A merchant buys 100 yards of cloth, at 15*s.* per yard; for what must he forthwith sell it, in order to gain 20 p. c. and allow 9 months' credit? *Ans.* £93. 7*s.* 6*d.*

53. A certain description of brass is made by fusing together 41 lbs. of old brass, 55 lbs. of refined copper, and 24 lbs. of zinc. What quantities, in the same proportion, must be fused, in order to produce a composition of 400 lbs., if 3 p. c. is allowed for waste?

Ans. 140·8935*lbs. B.*; 189·0034*lbs. C.*; &c.

54. A., B., and C., by ordinary exertion, can perform a work in 16, 18, and 21 days, respectively. In what time should it be accomplished by these three persons conjointly, if A. relaxes to $\frac{1}{2}$ his ordinary ability during $\frac{1}{2}$ his time,—B. exerts $\frac{1}{5}$ more than his ordinary ability, during $\frac{3}{4}$ of his time,—and C. works with his ordinary ability during his whole time? *Ans.* $6\frac{1}{3}\frac{0}{1}\frac{9}{5}\frac{3}{3}da.$

55. A room is 52 ft. 6 in. long, and 28 ft. 8 in. wide. Required the dimensions of another room equal in area, but having its length exactly double its width; also, how many planks, 22 ft. 2 in. by 7$\frac{1}{2}$ in. would floor both these rooms. *Ans.* 54·8634*ft. by* 27·4317*ft.*; 217·26*pl.*

56. I have just bought goods for £50., and am allowed 9 months' credit. If I now sell them for £50., with 3 months' credit, what is my present gain per cent.?

Ans. $2\frac{3}{7}\frac{3}{1}p.c.$

57. If 8*s.* 2*d.* worth of bread be consumed, in 7 days, by a family consisting of 6 adults and 5 children, when the price of the quartern loaf is 7$\frac{1}{2}$*d.*; how many of the adults, along with 3 of the children, will consume 12*s.* 9*d.* worth of bread in 15 days, when the price of the quartern loaf is 8$\frac{1}{2}$*d.*; supposing that 3 children consume as much, per day, as 2 adults? *Ans.* 4*ad.*

58. I bought 75 bushels of wheat, barley, and oats, for £18. For every 4 bushels of oats there were 6 of barley and 5 of wheat; also, the wheat, per bushel, was 2 prices of the barley, and 2$\frac{1}{2}$ of the oats. How many bushels of each grain did I buy, and at what prices?

Ans. 20*bu. oats, at* 3*s.*; &c.

59. I paid £75., a month ago, for a pipe of wine. How should I now sell it, at 3 months, to gain $12\frac{1}{2}$ p. c. on what it now costs me? *Ans.* £85. 15*s.* $8\frac{1}{2}d.\frac{7}{8}$.

60. I purchased goods for £23. at 12 months' credit, and thereupon sold them for £23. with such allowance of credit as made my immediate gain $1\frac{2}{71}$ p. c. How long credit did I give? *Ans.* $8\frac{1}{2}$ *mths.*

61. Find four numbers, such, that the first may be $\frac{2}{3}$ of the fourth, the second $\frac{3}{5}$ of the sum of the first and fourth, the third $\frac{4}{7}$ of the sum of the first, second, and fourth, and so that the fourth, added to $\frac{1}{2}$ of the sum of the first, second, and third, may amount to 281.

Ans. $83\frac{1}{3}$, $116\frac{2}{3}$; &c.

62. A., B., and C. traded, for one year, with a joint stock of £4500.,—A. investing £2000., B. £1500., and C. £1000. The stocks of B. and C. were sums borrowed from A. at $4\frac{1}{2}$ per cent. per annum; and C., as acting partner, was allowed $1\frac{1}{2}$ p. c. on the proceeds. At the end of the year their stock amounted to £5680. How much did C. then give to each of his partners, so as to fully settle the business?

Ans. To A. £5099. 1*s.* $6\frac{2}{3}d.$; B. £297. 8*s.* 8*d.*

63. I purchase wine-glasses at 8*s.* the dozen; at what price do I retail them, when the money I get for 10 dozen is equal to my gain on £100.? *Ans.* 8*s.* 4*d.*

64. A., B., and C. had £1000. of joint stock in trade. A.'s time was 7 months, B.'s 9, C.'s 10. In the division of the gain, B. got £7. for every £4. given to A., and £5. for every £6. given to C. Find the several stocks.

Ans. A.'s £261·0209.; B.'s £355·2784.; &c.

65. A., B., and C. jointly perform a work, which they could severally do in 10, 9, and 12 days, respectively. If B. should begin 2 days later than A., and C. 1 day later than B., in what time would the work be performed? *Ans.* 5*da.*

66. If I sell the gallon of spirits at 16*s.* 6*d.* ready money, I shall lose 1 p. c.:—If I sell it at 17*s.*, what term of credit must I allow, so that I may neither lose nor gain? Ans. $4\frac{4}{17}$mths.

67. A., B. and C. form a joint stock,—the particulars being A. £340.; B. £280.; C. £255. B.'s time is $3\frac{1}{3}$ months longer than C.'s; also, A.'s gain is £55. 5*s.*, B.'s £59. 10*s.*, and C.'s £38. 5*s.* Required the particular times. *Ans.* A. $8\frac{1}{3}$, B. $11\frac{1}{3}$, C. 8*mths.*

TABLES

OF MONEY, WEIGHT, AND MEASURE.

MONEY.

2 farthings = 1 halfpenny.	5 shillings = 1 crown.
4 farthings = 1 penny. (*d.*)	20 shillings = 1 pound. (£.)
12 pence .. = 1 shilling. (*s.*)	21 shillings = 1 guinea.

One farthing is marked ¼*d.*; *two farthings*, or *one halfpenny*, ½*d.*, *three farthings*, ¾*d.*

TROY WEIGHT.

24 grains..... = 1 pennyweight.	(*dwt.*)
20 pennyweights ..... = 1 ounce.	(*oz.*)
12 ounces..... = 1 pound.	(*lb.*)

Troy Weight is used in weighing *gold, silver, jewels, &c.*

AVOIRDUPOIS WEIGHT.

16 drams..... = 1 ounce.	(*oz.*)
16 ounces ..... = 1 pound.	(*lb.*)
28 pounds..... = 1 quarter.	(*qr.*)
4 quarters, or 112 lbs..... = 1 hundredweight.	(*cwt.*)
20 hundredweights ..... = 1 ton.	(*t.*)

LINEAL OR LONG MEASURE.

12 inches ..... = 1 foot.
3 feet..... = 1 yard.
5½ yards..... = 1 pole.
40 poles, or 220 yards ..... = 1 furlong.
8 furlongs, or 1760 yards..... = 1 mile.

CLOTH MEASURE.

2¼ inches ..... = 1 nail.
4 nails, or 9 inches ..... = 1 quarter.
4 quarters..... = 1 yard.
5 qrs. of a yd. ..... = 1 English ell.
3 qrs. of a yd. ..... = 1 Flemish ell.
6 qrs. of a yd. ..... = 1 French ell.

SUPERFICIAL OR SQUARE MEASURE.

144 square inches ..... = 1 square foot.
9 square feet ..... = 1 square yard.
30¼ square yards ..... = 1 square pole.
40 square poles ..... = 1 rood.
4 roods ..... = 1 acre.
640 acres ..... = 1 square mile.

SOLID OR CUBIC MEASURE.

1728 cubic inches ..... = 1 cubic foot.
27 cubic feet ..... = 1 cubic yard.

MEASURE OF CAPACITY.

2 pints ..... = 1 quart.
4 quarts ..... = 1 gallon.
2 gallons ..... = 1 peck.
4 pecks ..... = 1 bushel.
8 bushels ..... = 1 quarter.

The *peck*, *bushel*, and *quarter*, are used only for *dry goods*.

TIME.

60 seconds = 1 minute.
60 minutes = 1 hour.
24 hours .. = 1 day.
7 days = 1 week.
365 days = 1 common year.
366 days = 1 leap year.

CALENDAR MONTHS.

	Days.		Days.		Days.
January	31	May	31	September	30
February	28	June	30	October	31
March	31	July	31	November	30
April	30	August	31	December	31

Each month has 31 days, excepting February, which has 28, and in leap-year 29; excepting also the months mentioned in the lines—

Thirty days have September,
April, June, and November.

SIGNS OF OPERATION.

+, named *plus*, denotes Addition.
—, named *minus*, denotes Subtraction.
× denotes Multiplication.
÷ denotes Division.
= denotes Equality.

NOTES ON THE TABLES OF MONEY, WEIGHT, AND MEASURE.

MONEY.—The correct weight of the silver penny, in the Saxon times, was the 240th of the Saxon Pound (about 1-16th less than the present Troy pound); hence the 240th part of the Troy Pound is called a *Pennyweight*. The Pound weight of silver was coined into 20 shillings; and the name *Pound* has been retained to denote the value of 20 shillings, although the value of the shilling has been much diminished.

As the British nation has chosen to have gold for its standard of value, the copper coinage is not considered a legal tender beyond 12*d*., nor the silver above 40*s*., for one payment.

The standard gold coin of this country consists of 22 parts of pure gold with two parts of copper alloy; this forms the arbitrary *assay* pound of 24 *carats*, each 4 grains; and hence standard gold is said to be 22 carats *fine*.

The standard silver coin contains 37 parts of pure silver with 3 parts of copper alloy. Hence a lb. Troy of standard silver contains 222 dwts. of pure silver and 18 dwts. of alloy, and standard silver is said to be 11 ozs. 2 dwts. *fine*.

The mintage rate of standard gold in Britain is 77*s*. 10½*d*. per ounce, and of standard silver 5*s*. 6*d*. per ounce. The Avoirdupois lb. of copper is coined into 24 pence.

The initial letters of the Latin words *Libra*, *Solidus*, *Denarius*, and *Quadrans*, are symbols for the meanings of these terms, denoting *Pound*, *Shilling*, *Penny*, and *Farthing* respectively; but 1, 2, and 3 farthings are now represented by fractional parts of a penny: thus, instead of 12*s*. 5*d*. 2*q*., we write 12*s*. 5½*d*.

The following coins are occasionally referred to, though now obsolete:—A Groat = 4*d*.; A Noble = 6*s*. 8*d*.; An Angel = 10*s*.; A Mark = 13*s*. 4*d*.; A Guinea = 21*s*.

TROY WEIGHT.—*Troy* and *Avoirdupois* are the only Imperial weights recognised by Act of Parliament.

The term *Troy* Weight was derived, as some say, from *Troyes*, the name of a town in France; but others refer to *Troy-novant*, the monkish name of London, as a more likely origin.

A cubic inch of distilled water, of the temperature of 62° Fahrenheit, when the barometer is at 30°, is understood to weigh 252·458 grains, which is the parliamentary standard for determining the weight of the Troy grain.

Apothecary's Weight, which differs from Troy weight only in the subdivisions of the ounce, is allowed, but not enforced, by Government. It is employed for medical prescriptions; the ounce Troy being divided into 8 drams, the dram into 3 scruples, and the scruple into 20 grains.

AVOIRDUPOIS WEIGHT.—The name Avoirdupois is with great probability traced to the Norman, *avoirs*, goods and chattels, and *poids*, weight.

7000 Troy grains make 1 lb. Avoirdupois; and as the lb. Troy contains 5760 grains, the lb. Troy is to the lb. Avoirdupois as 5760 to 7000, or as 144 to 175. Also, 480 grains make 1 oz. Troy, and 437½ grains make 1 oz. Avoirdupois; hence, the Troy oz. is to the Avoirdupois oz. as 960 to 875, or as 192 to 175.

LINEAL MEASURE.—12 lines = 1 inch; 6 feet = 1 fathom; 69⅔ English miles = 60 nautical or geographical miles; hence a knot or nautical mile = 2025⅓ yards; 3 nautical miles = 1 league.

In Gunter's chain, used for land measuring, 7·92 inches = 1 link; 4 poles or 66 feet = 1 chain of 100 links; 80 chains = 1 mile.

The length of a pendulum vibrating seconds, in the latitude of London, is 39·1393 inches. This is the standard of lineal measure.

SUPERFICIAL MEASURE.—10 chains in length by 1 in breadth = 10 square chains = 100,000 square links = 1 acre.

SOLID MEASURE.—A cubic foot of distilled water, at the temperature of 62°, weighs about 1000 ounces Avoirdupois = 62½ lbs.; and this forms the basis of a Table of Specific Gravities. Thus, a cubic foot of mahogany weighs 1063 ounces, and its specific gravity is 1·063, *i. e.* its weight is 1·063 as great as that of an equal bulk of water.

MEASURE OF CAPACITY.—5 Avoirdupois ounces of water, at 62°, = 1 gill; 4 gills = 1 pint; 10 lbs. Avoirdupois of water = 1 gallon = 277·274 cubic inches.

TIME.—In the time of Julius Cæsar the year was understood to consist of 365 da. 6 hrs., or 365¼ da. Therefore, according to the Julian Calendar, every 4th year is made to consist of 366 days, and each of the three preceding or following has 365. The year of 366 days is called *Leap* year, and is known by its number being exactly divisible by 4; thus A.D. 1851 is the 3rd year following a leap year; 1852 is a leap year. The leap year used to be called *Bissextile*, from two Latin words signifying *twice the sixth*, because the 6th day before the *Calends* or commencement of March, in the Roman Calendar, was repeated to introduce the additional or *intercalary* day. We add the leap-*year day* at the close of February.

The time, however, between the sun's leaving one point in the Ecliptic and returning to that point is 365 da. 5 ho. 48 min. 50 sec., which is therefore the true solar year;* and as this is 11 min. 10 sec. less than the Julian Account, the error of the latter will amount to about 3 days in 400 years. In rectification of this, Pope Gregory XIII., in the year 1582, ordained that the 5th of October should be reckoned as the 15th, and that thereafter, in every 400 years, 3 leap-year days should be suppressed, by accounting any year which denotes an exact number of centuries to be leap-year only when the number of the century should be exactly divisible by 4. By this means 1600 and 2000 are made leap years; whereas 1700, 1800, and 1900, are made common years, although according to the Julian scheme they would be leap years.

The Gregorian Calendar was adopted in England in 1752, when the error had amounted to 11 days; and then the day following the 2nd of September was reckoned as the 14th. The *New Style*, as the Gregorian scheme is called, is now 12 days before the *Old* or Julian *Style*.

ANGULAR MEASURE.—The circumference of every circle is supposed to be divided into 360 degrees (360°), each degree into 60 minutes (60′), and each minute into 60 seconds (60″).

The French divide the circle differently, making the *grade*, or degree, the 400th part of a circle, the minute the 100th part of a degree, and the second the 100th part of a minute.

SIGNS OF OPERATION.—The principal of these are +, −, ×, ÷, ∴, =, √, and ( ).

+, the sign of Addition, appears to have been originally the copulative conjunction &, a contraction of the Latin word *et*, being probably a form resulting from rapidity in writing the original character.

−, the sign of Subtraction, is probably the result of a rapid formation of the letter *s* placed horizontally between the minuend and subtrahend; thus, 5 ⌒ 3, *i. e.* 5 *subductis* 3, or 5 *subtracting* 3, might come to be represented by 5 − 3, just as 5 shillings are

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* The *Sidereal* year of the earth, or that time which the sun takes in moving through the Ecliptic from any fixed star to the same star again, is 365 da. 6 ho. 9 min. 10 sec. This contains an excess of 20 min. 20 sec. above the *Tropical* year, occasioned by the Precession of the Equinoxes, a slow retrograde motion of the Equinoctial Points, amounting to about 50·1 seconds of a degree in a year.

360° : 50·1″ : : 365 da. 6 ho. 9 min. 10 sec. : 20 min. 20 sec.

now frequently represented by 5/. Algebraists employ the Italic *f* horizontally, to denote subtraction, when it is not presumed which of two quantities is the minuend.

The signs + and − were first introduced in publication by Stifel, in 1544.

×, the sign of Multiplication, is said to have been first employed by Oughtred, in his *Clavis Mathematica*, 1631. It had been used, however, much earlier, as a sign of reciprocal multiplication, in the reduction of fractions to a common denominator, &c.* Thus, $\frac{2}{3} \times \frac{4}{5}$ originally signified the conversion of the numerators into 14 and 20, the form of the symbol being a natural indication of the process.

÷, the sign of Division, was derived from the Hindu practice of writing a dividend over its divisor.

The retention of dividend and divisor, to express Ratio, is represented by withdrawing the line from the points; thus, 16 ∶ 8 signifies $\frac{2}{1}$. Similarly, a double sign of division, ⁝÷⁝, having the line withdrawn, denotes Proportion, or Equality of Ratios; thus 4 : 3 : : 9 : $6\frac{3}{4}$ denotes that $4 \div 3$ equals $9 \div 6\frac{3}{4}$. This use of the points was introduced by Oughtred.

=, the sign of Equality, was devised by Robert Recorde, who, in his *Whetstone of Witte* (the first English work on Algebra, 1557), tells us that two equal and parallel lines appeared to him the most natural symbol of equality.

√, the sign of Evolution, is a form of the initial letter of the Latin word *radix*, signifying *root*.

( ), the sign of Collection, denotes that the sign preceding or following it has reference to the collected result of the quantities enclosed; sometimes a horizontal line, called a Vinculum (the Latin word for a *bond* or *tie*), is placed over the quantities to be collected. Thus, $\sqrt{(16 + 9)}$, or $\sqrt{16 + 9}$, is equal to 5; whereas $\sqrt{16} + 9$ denotes 13.

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* This use of the sign is called by Bp. Tonstall 'multiplicatio in speciem crucis divi Andreæ.' *De Arte Supputandi*. Paris, 1529.

THE END.

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